chapter 7

Portfolio Theory and Other Asset Pricing Models

Americans love mutual funds. By 1985, they had invested about $495 billion in mutual funds, which is not exactly chicken feed. By mid-2006, however, they had invested more than $9 trillion in mutual funds! Not only has the amount of money invested in mutual funds skyrocketed, but the variety of funds is astounding. You can buy funds that specialize in virtually any type of asset, funds that specialize in stocks from a particular industry, a particular continent, or a particular country. There are money market funds that invest only in Treasury bills and other short-term securities, and there are even funds that hold municipal bonds from a specific state.

For those of you with a social conscience, you can buy funds that refuse to own stocks of companies that pollute, sell tobacco products, or have workforces that are not culturally diverse. For others, there is the "Vice Fund," which invests only in brewers, defense contractors, tobacco companies, and the like.

You can also buy "market neutral funds," which sell some stocks short, invest in others, and promise (perhaps falsely) to do well no matter which way the market goes. There is the Undiscovered Managers Behavioral fund that picks stocks by psychoanalyzing Wall Street analysts. And then there is the Tombstone fund, which owns stocks only from the funeral industry.

You can buy an index fund, which simply holds a portfolio of stocks in an index such as the S&P 500 and doesn't try to beat the market. Instead, index funds strive for low expenses and pass the savings on to investors. An Exchange Traded Fund, or ETF, actually has its own stock that is traded on a stock exchange. Different ETFs hold widely varied portfolios, ranging from the S&P 500 to gold mining companies to Middle Eastern oil companies, and their fees to long-term investors are quite low. At the other extreme, hedge funds, which are pools of money provided by institutions and wealthy individuals, are extremely actively managed—even to the extent of taking over and then operationally managing firms in the portfolio—and have relatively high expenses.

As you read this chapter, think about how portfolio theory, which became widely understood about 30 years ago, has influenced the mutual fund industry.

Sources: The Securities Industry and Financial Markets Association, June 2006, **http://www.sia.com/research/pdf/keystats.pdf;** "The Many New Faces of Mutual Funds," *Fortune*, July 6, 1998, pp. 217–218; and "Street Myths," *Fortune*, May 24, 1999, p. 320.

In Chapter 6 we presented the key elements of risk and return analysis. There we saw that much of a stock's risk can be eliminated by diversification, so rational investors should hold portfolios of stocks rather than just one stock. We also introduced the Capital Asset Pricing Model (CAPM), which links risk and required rates of return, using a stock's beta coefficient as the relevant measure of risk. In this chapter, we extend these concepts and explain portfolio theory. We then present an in-depth treatment of the CAPM, including a more detailed look at how betas are calculated. We discuss two other asset pricing models, the Arbitrage Pricing Theory model and the Fama-French three-factor model. Last, we introduce a new but fast-growing field, behavioral finance.

e-resource

The textbook's Web site contains an *Excel* file that will guide you through the chapter's calculations. The file for this chapter is ***FM12 Ch 07 Tool Kit.xls,*** and we encourage you to open the file and follow along as you read the chapter.

7.1 Efficient Portfolios

Recall from Chapter 6 the important role that the correlation between assets plays in portfolio risk. One important use of portfolio risk concepts is to select **efficient portfolios,** defined as those portfolios that provide the highest expected return for any degree of risk, or the lowest degree of risk for any expected return. We begin with the two-asset case and then extend it to the general case of N assets.

The Two-Asset Case

Consider two assets, A and B. Suppose we have estimated the expected returns ($\hat{r}_A$ and $\hat{r}_B$), the standard deviations (σ_A and σ_B) of returns, and the correlation coefficient (ρ_{AB}) for returns.[1] The expected return and standard deviation for a portfolio containing these two assets are

$$\hat{r}_p = w_A\hat{r}_A + (1 - w_A)\hat{r}_B \tag{7-1}$$

and

$$\text{Portfolio SD} = \sigma_p = \sqrt{w_A^2\sigma_A^2 + (1 - w_A)^2\sigma_B^2 + 2w_A(1 - w_A)\rho_{AB}\sigma_A\sigma_B}. \tag{7-2}$$

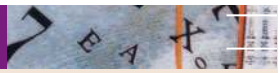
Here w_A is the fraction of the portfolio invested in Security A, so $(1 - w_A)$ is the fraction invested in Security B.

To illustrate, suppose we can allocate our funds between A and B in any proportion. Suppose Security A has an expected rate of return of $\hat{r}_A = 5\%$ and a standard deviation of returns $\sigma_A = 4\%$, while $\hat{r}_B = 8\%$ and $\sigma_B = 10\%$. Our first task is to determine the set of *attainable* portfolios, and then from this attainable set to select the *efficient* subset.

To construct the attainable set, we need data on the degree of correlation between the two securities' expected returns, ρ_{AB}. Let us work with three different assumed degrees of correlation, $\rho_{AB} = +1.0$, $\rho_{AB} = 0$, and $\rho_{AB} = -1.0$, and use

[1] See Chapter 6 for definitions using historical data to estimate the expected return, standard deviation, covariance, and correlation.

Corporate Valuation and Risk



In Chapter 1, we told you that managers should strive to make their firms more valuable and that the value of a firm is determined by the size, timing, and risk of its free cash flows (FCF). This chapter provides additional insights into how to measure a firm's risk, which affects its WACC and its value.

$$\text{Value} = \frac{FCF_1}{(1 + WACC)^1} + \frac{FCF_2}{(1 + WACC)^2} + \frac{FCF_3}{(1 + WACC)^3} + \cdots + \frac{FCF_\infty}{(1 + WACC)^\infty}$$

them to develop the portfolios' expected returns, $\hat{r}_p$, and standard deviations, σ_p. (Of course, only one correlation can exist; our example simply shows three alternative situations that might exist.)

To calculate $\hat{r}_p$, we use Equation 7-1, substituting the given values for $\hat{r}_A$ and $\hat{r}_B$, and then calculating $\hat{r}_p$ for different values of w_A. For example, when w_A equals 0.75, then $\hat{r}_p = 5.75\%$:

$$\begin{aligned}\hat{r}_p &= w_A\hat{r}_A + (1 - w_A)\hat{r}_B \\ &= 0.75(5\%) + 0.25(8\%) = 5.75\%.\end{aligned}$$

Other values of $\hat{r}_p$ were found similarly, and they are shown in the $\hat{r}_p$ column of Table 7-1.

Next, we use Equation 7-2 to find σ_p. Substitute the given values for σ_A, σ_B, and ρ_{AB}, and then calculate σ_p for different values of w_A. For example, in the case where $\rho_{AB} = 0$ and $w_A = 0.75$, then $\sigma_p = 3.9\%$:

$$\begin{aligned}\sigma_p &= \sqrt{w_A^2\sigma_A^2 + (1 - w_A)^2\sigma_B^2 + 2w_A(1 - w_A)\rho_{AB}\sigma_A\sigma_B} \\ &= \sqrt{(0.5625)(0.0016) + (0.0625)(0.01) + 2(0.75)(0.25)(0)(0.04)(0.10)} \\ &= \sqrt{0.0009 + 0.000625} = \sqrt{0.001525} = 0.039 = 3.9\%.\end{aligned}$$

Table 7-1 $\hat{r}_p$ and σ_p under Various Assumptions

Proportion of Portfolio in Security A (Value of w_A)	Proportion of Portfolio in Security B (Value of $1 - w_A$)	$\hat{r}_p$	σ_p		
			Case I ($\rho_{AB} = +1.0$)	Case II ($\rho_{AB} = 0$)	Case III ($\rho_{AB} = -1.0$)
1.00	0.00	5.00%	4.0%	4.0%	4.0%
0.75	0.25	5.75	5.5	3.9	0.5
0.50	0.50	6.50	7.0	5.4	3.0
0.25	0.75	7.25	8.5	7.6	6.5
0.00	1.00	8.00	10.0	10.0	10.0

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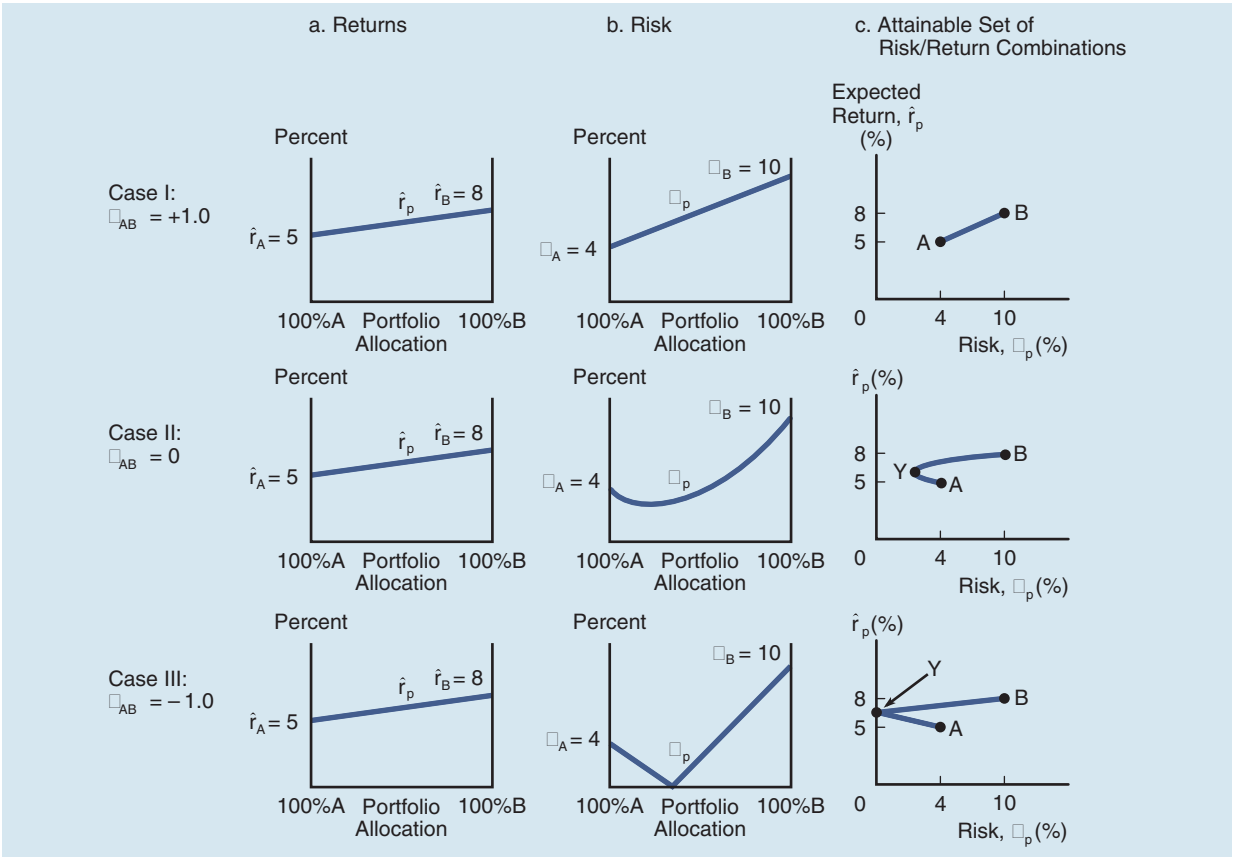
See ***FM12 Ch 07 Tool Kit.xls*** at the textbook's Web site for all calculations.

Table 7-1 gives $\hat{r}_p$ and σ_p values for $w_A = 1.00, 0.75, 0.50, 0.25$, and 0.00, and Figure 7-1 plots $\hat{r}_p$, σ_p, and the attainable set of portfolios for each correlation. In both the table and the graphs, note the following points:

1. The three graphs across the top row of Figure 7-1 designate Case I, where the two assets are perfectly positively correlated; that is, $\rho_{AB} = +1.0$. The three graphs in the middle row are for the zero correlation case, and the three in the bottom row are for perfect negative correlation.
2. We rarely encounter $\rho_{AB} = -1.0$, 0.0, or $+1.0$. Generally, ρ_{AB} is in the range of $+0.5$ to $+0.7$ for most stocks. Case II (zero correlation) produces graphs which, pictorially, most closely resemble real-world examples.
3. The left column of graphs shows how the *expected portfolio returns* vary with different combinations of A and B. We see that these graphs are identical in each of the three cases: The portfolio return, $\hat{r}_p$, is a linear function of w_A, and it does not depend on the correlation coefficients. This is also seen from the single $\hat{r}_p$ column in Table 7-1.
4. The middle column of graphs shows how risk is affected by the portfolio mix. Starting from the top, we see that portfolio risk, σ_p, increases linearly in Case I, where $\rho_{AB} = +1.0$; it is nonlinear in Case II; and Case III shows that risk can be completely diversified away if $\rho_{AB} = -1.0$. Thus σ_p, unlike $\hat{r}_p$, *does* depend on correlation.

Figure 7-1 Illustrations of Portfolio Returns, Risk, and the Attainable Set of Portfolios



5. Note that in both Cases II and III, but not in Case I, someone holding only Stock A could sell some A, buy some B, and both increase his or her expected return and lower risk.
6. The right column of graphs shows the attainable, or feasible, set of portfolios constructed with different mixes of Securities A and B. Unlike the other columns, which plotted return and risk versus the portfolio's composition, each of the three graphs here was plotted from pairs of $\hat{r}_p$ and σ_p as shown in Table 7-1. For example, Point A in the upper right graph is the point $\hat{r}_p = 5\%$, $\sigma_p = 4\%$ from the Case I data. All other points on the curves were plotted similarly. With only two securities in the portfolio, the attainable set is a curve or line, and we can achieve each risk/return combination on the relevant curve by some allocation of our investment funds between Securities A and B.
7. Are all combinations on the attainable set equally good? The answer is no. Only that part of the attainable set from Y to B in Cases II and III is defined to be efficient. The part from A to Y is inefficient because for any degree of risk on the line segment AY, a higher return can be found on segment YB. Thus, no rational investor would hold a portfolio that lies on segment AY. In Case I, however, the entire feasible set is efficient—no combination of the securities can be ruled out.

From these examples we see that in one extreme case ($\rho = -1.0$), risk can be completely eliminated, while in the other extreme case ($\rho = +1.0$), diversification does no good whatsoever. In between these extremes, combining two stocks into a portfolio reduces but does not eliminate the risk inherent in the individual stocks. If we differentiate Equation 7-2, set the derivative equal to zero, and then solve for w_A, we obtain the fraction of the portfolio that should be invested in Security A if we wish to form the least-risky portfolio. Here is the equation:

$$\text{Minimum risk portfolio: } w_A = \frac{\sigma_B(\sigma_B - \rho_{AB}\sigma_A)}{\sigma_A^2 + \sigma_B^2 - 2\rho_{AB}\sigma_A\sigma_B}. \qquad (7\text{-}3)$$

As a rule, we limit w_A to the range 0 to +1.0; that is, if the solution value is $w_A > 1.0$, set $w_A = 1.0$, and if w_A is negative, set $w_A = 0.0$. A w_A value that is negative means that Security A is sold short; if w_A is greater than 1.0, B is sold short. In a short sale, you borrow a stock and then sell it, expecting to buy it back later (at a lower price) in order to repay the person from whom the stock was borrowed. If you sell short and the stock price rises, you lose, but you win if the price declines.

The N-Asset Case

The same principles from the two-asset case also apply when the portfolio is composed of N assets. Here is the notation for the N-asset case: The percentage of the investment in asset i (the portfolio weight) is w_i, the expected return for asset i is $\hat{r}_i$, the standard deviation of asset i is σ_i, and the correlation between asset i and asset j is ρ_{ij}. The expected return for a portfolio with N assets is

$$\hat{r}_p = \sum_{i=1}^{N} (w_i \hat{r}_i) \qquad (7\text{-}4)$$

and the variance of the portfolio is

$$\sigma_p^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} (w_i w_j \sigma_i \sigma_j \rho_{ij}). \quad (7\text{-}5)$$

For the case in which i = j, the correlation is $\rho_{ii} = 1$. Notice also that when i = j, the product $\sigma_i \sigma_i = \sigma_i^2$.

One way to apply Equation 7-5 is to set up a table, with a row and column for each asset. Give the rows and columns labels showing the assets' weights and standard deviations. Then fill in each cell in the table by multiplying the values in the row and column headings by the correlation between the assets, as shown below:

	$w_1\sigma_1$ (1)	$w_2\sigma_2$ (2)	$w_3\sigma_3$ (3)
$w_1\sigma_1$ (1)	$w_1\sigma_1 w_1\sigma_1\rho_{11} = w_1^2\sigma_1^2$	$w_1\sigma_1 w_2\sigma_2\rho_{12}$	$w_1\sigma_1 w_3\sigma_3\rho_{13}$
$w_2\sigma_2$ (2)	$w_2\sigma_2 w_1\sigma_1\rho_{21}$	$w_2\sigma_2 w_2\sigma_2\rho_{22} = w_2^2\sigma_2^2$	$w_2\sigma_2 w_3\sigma_3\rho_{23}$
$w_3\sigma_3$ (3)	$w_3\sigma_3 w_1\sigma_1\rho_{31}$	$w_3\sigma_3 w_2\sigma_2\rho_{32}$	$w_3\sigma_3 w_3\sigma_3\rho_{33} = w_3^2\sigma_3^2$

The portfolio variance is the sum of the nine cells. For the diagonals, we have substituted the values for the case in which i = j. Notice that some of the cells have identical values. For example, the cell for Row 1 and Column 2 has the same value as the cell for Column 1 and Row 2. This suggests an alternative formula:

$$\sigma_p^2 = \sum_{i=1}^{N} w_i^2\sigma_i^2 + \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} 2 w_i \sigma_i w_j \sigma_j \rho_{ij}. \quad (7\text{-}5a)$$

The main thing to remember when calculating portfolio standard deviations is to not leave out any terms. Using a table like the one above can help.

SELF-TEST

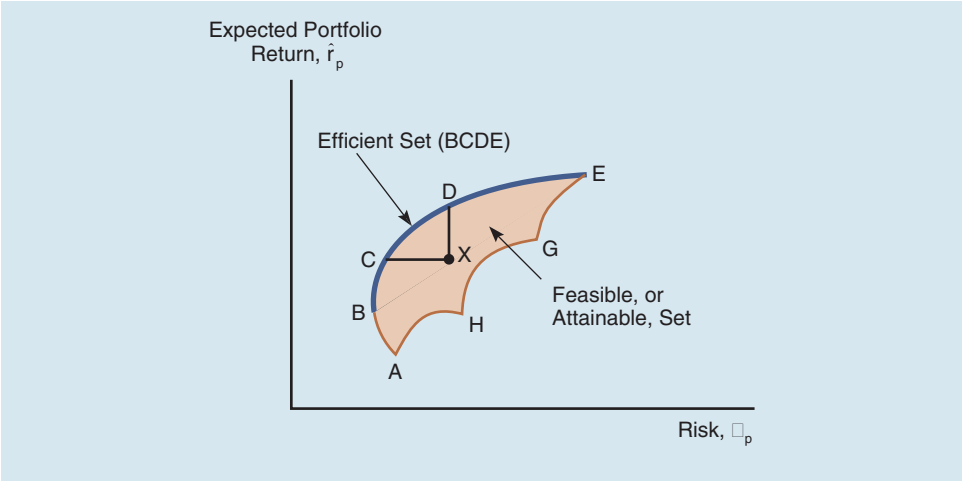
What is meant by the term "attainable, or feasible, set"?
Within the attainable set, which portfolios are "efficient"?
Stock A has an expected return of 10% and a standard deviation of 35%. Stock B has an expected return of 14% and a standard deviation of 45%. The correlation coefficient between Stock A and B is 0.3. What are the expected return and standard deviation of a portfolio invested 60% in Stock A and 40% in Stock B? (12.0%; 31.5%)

7.2 Choosing the Optimal Portfolio

With only two assets, the feasible set of portfolios is a line or curve as shown in the third column of graphs back in Figure 7-1. However, if we were to increase the number of assets, we would obtain an area like the shaded area in Figure 7-2. The points A, H, G, and E represent single securities (or portfolios containing only one security). All the other points in the shaded area and its boundaries, which comprise the feasible set, represent portfolios of two or more securities. Each point in this area represents a particular portfolio with a risk of σ_p and an expected return of $\hat{r}_p$. For example, point X represents one such portfolio's risk and expected return, as do B, C, and D.

Figure 7-2

The Efficient Set of Investments



Given the full set of potential portfolios that could be constructed from the available assets, which portfolio should actually be held? This choice involves two separate decisions: (1) determining the efficient set of portfolios and (2) choosing from the efficient set the single portfolio that is best for the specific investor.

The Efficient Frontier

In Figure 7-2, the boundary line BCDE defines the efficient set of portfolios, which is also called the **efficient frontier.**[2] Portfolios to the left of the efficient set are not possible because they lie outside the attainable set. Portfolios to the right of the boundary line (interior portfolios) are inefficient because some other portfolio would provide either a higher return for the same degree of risk or a lower risk for the same rate of return. For example, Portfolio X is dominated by Portfolios C and D.

Risk/Return Indifference Curves

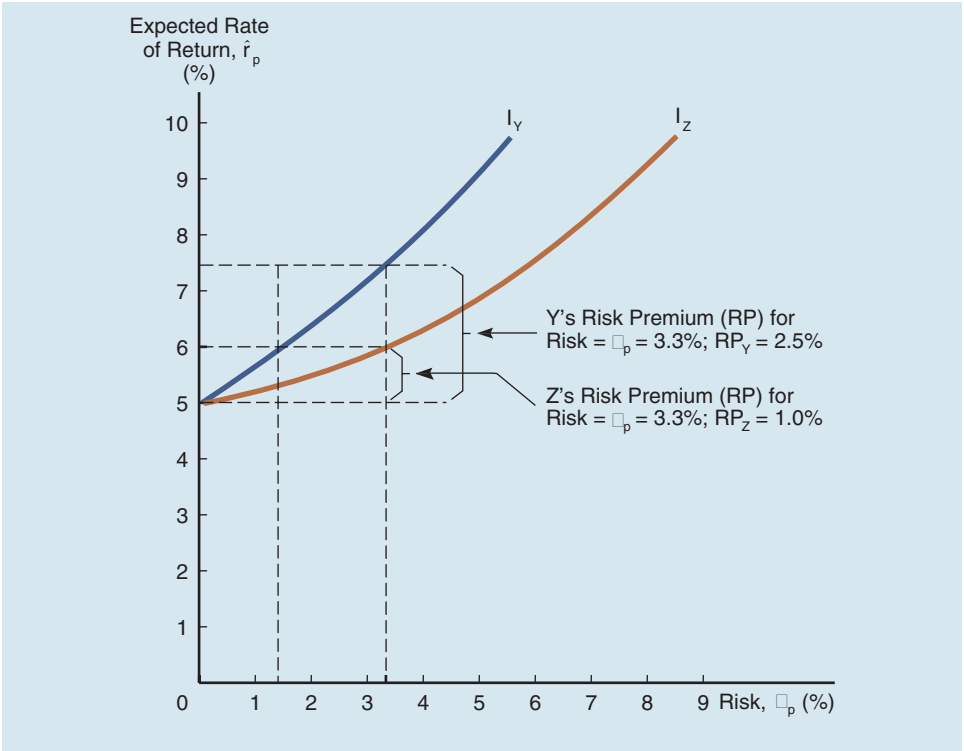
Given the efficient set of portfolios, which specific portfolio should an investor choose? To determine the optimal portfolio for a particular investor, we must know the investor's attitude toward risk as reflected in his or her risk/return trade-off function, or **indifference curve.**

An investor's risk/return trade-off function is based on the standard economic concepts of utility theory and indifference curves, which are illustrated in Figure 7-3. The curves labeled I_Y and I_Z represent the indifference curves of Individuals Y and Z. Ms. Y is indifferent between the riskless 5% portfolio, a portfolio with an expected return of 6% but a risk of $\sigma_p = 1.4\%$, and so on. Mr. Z is indifferent between a riskless 5% return, an expected 6% return with risk of $\sigma_p = 3.3\%$, and so on.

[2]A computational procedure for determining the efficient set of portfolios was developed by Harry Markowitz and first reported in his article "Portfolio Selection," *Journal of Finance*, March 1952. In this article, Markowitz developed the basic concepts of portfolio theory, and he later won the Nobel Prize in economics for his work.

Figure 7-3

Risk/Return Indifference Curves



Note that Ms. Y requires a higher expected rate of return as compensation for any given amount of risk; thus, Ms. Y is said to be more **risk averse** than Mr. Z. Her higher risk aversion causes Ms. Y to require a higher **risk premium**—defined here as the difference between the 5% riskless return and the expected return required to compensate for any specific amount of risk—than Mr. Z requires. Thus, Ms. Y requires a risk premium (RP_Y) of 2.5% to compensate for a risk of $\sigma_p = 3.3\%$, while Mr. Z's risk premium for this degree of risk is only $RP_Z = 1.0\%$. *As a generalization, the steeper the slope of an investor's indifference curve, the more risk averse the investor.* Thus, Ms. Y is more risk averse than Mr. Z.

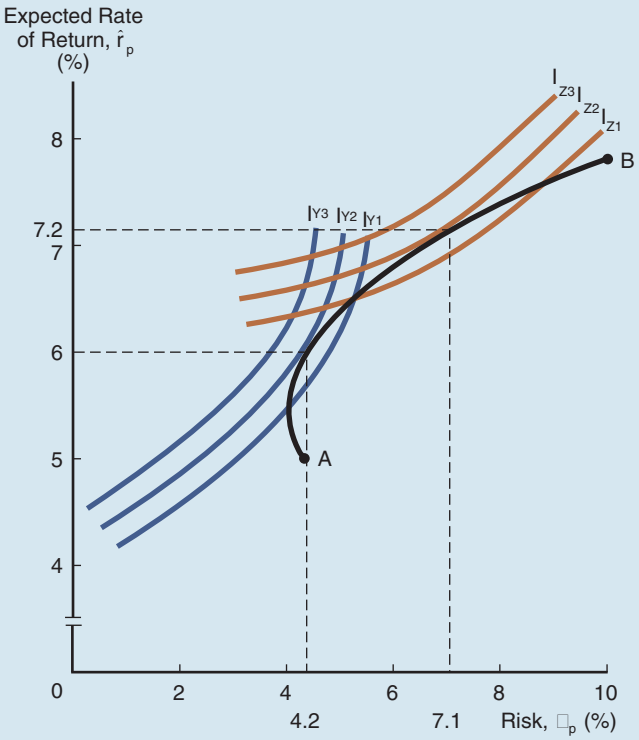
Each individual has a "map" of indifference curves; the indifference maps for Ms. Y and Mr. Z are shown in Figure 7-4. The higher curves denote a greater level of satisfaction (or utility). Thus, I_{Z2} is better than I_{Z1} because, for any level of risk, Mr. Z has a higher expected return, hence greater utility. An infinite number of indifference curves could be drawn in the map for each individual, and each individual has a unique map.

The Optimal Portfolio for an Investor

Figure 7-4 also shows the feasible set of portfolios for the two-asset case, under the assumption that $\rho_{AB} = 0$, as it was developed in Figure 7-1. The optimal portfolio for each investor is found at the tangency point between the efficient set of portfolios and one of the investor's indifference curves. This tangency point marks the highest level of satisfaction the investor can attain. Ms. Y, who is more risk averse than Mr. Z, chooses a portfolio with a lower expected return (about 6%) but a risk

Figure 7-4

Selecting the Optimal Portfolio of Risky Assets



of only $\sigma_p = 4.2\%$. Mr. Z picks a portfolio that provides an expected return of about 7.2%, but it has a risk of about $\sigma_p = 7.1\%$. Ms. Y's portfolio is more heavily weighted with the less risky security, while Mr. Z's portfolio contains a larger proportion of the more risky security.[3]

SELF-TEST

What is the efficient frontier?
What are indifference curves?
Conceptually, how does an investor choose his or her optimal portfolio?

7.3 The Basic Assumptions of the Capital Asset Pricing Model

The **Capital Asset Pricing Model (CAPM),** which was introduced in the last chapter, specifies the relationship between risk and required rates of return on assets

[3]Ms. Y's portfolio would contain 67% of Security A and 33% of Security B, whereas Mr. Z's portfolio would consist of 27% of Security A and 73% of Security B. These percentages can be determined with Equation 7-1 by simply seeing what percentage of the two securities is consistent with $\hat{r}_p = 6.0\%$ and 7.2%. For example, $w_A(5\%) + (1 - w_A)(8\%) = 7.2\%$, and solving for w_A, we obtain $w_A = 0.27$ and $(1 - w_A) = 0.73$.

when they are held in well-diversified portfolios. The assumptions underlying the CAPM's development are summarized in the following list:[4]

1. All investors focus on a single holding period, and they seek to maximize the expected utility of their terminal wealth by choosing among alternative portfolios on the basis of each portfolio's expected return and standard deviation.
2. All investors can borrow or lend an unlimited amount at a given risk-free rate of interest, r_{RF}, and there are no restrictions on short sales of any asset.[5]
3. All investors have identical estimates of the expected returns, variances, and covariances among all assets (that is, investors have homogeneous expectations).
4. All assets are perfectly divisible and perfectly liquid (that is, marketable at the going price).
5. There are no transactions costs.
6. There are no taxes.
7. All investors are price takers (that is, all investors assume that their own buying and selling activity will not affect stock prices).
8. The quantities of all assets are given and fixed.

Theoretical extensions in the literature have relaxed some of these assumptions, and in general these extensions have led to conclusions that are reasonably consistent with the basic theory. However, the validity of any model can be established only through empirical tests, which we discuss later in the chapter.

SELF-TEST

What are the key assumptions of the CAPM?

7.4 The Capital Market Line and the Security Market Line

Figure 7-4 showed the set of portfolio opportunities for the two-asset case, and it illustrated how indifference curves can be used to select the optimal portfolio from the feasible set. In Figure 7-5, we show a similar diagram for the many-asset case, but here we also include a risk-free asset with a return r_{RF}. The riskless asset by definition has zero risk, $\sigma = 0\%$, so it is plotted on the vertical axis.

The figure shows both the feasible set of portfolios of risky assets (the shaded area) and a set of indifference curves (I_1, I_2, I_3) for a particular investor. Point N, where indifference curve I_1 is tangent to the efficient set, represents a possible portfolio choice; it is the point on the efficient set of risky portfolios where the investor obtains the highest possible return for a given amount of risk and the smallest degree of risk for a given expected return.

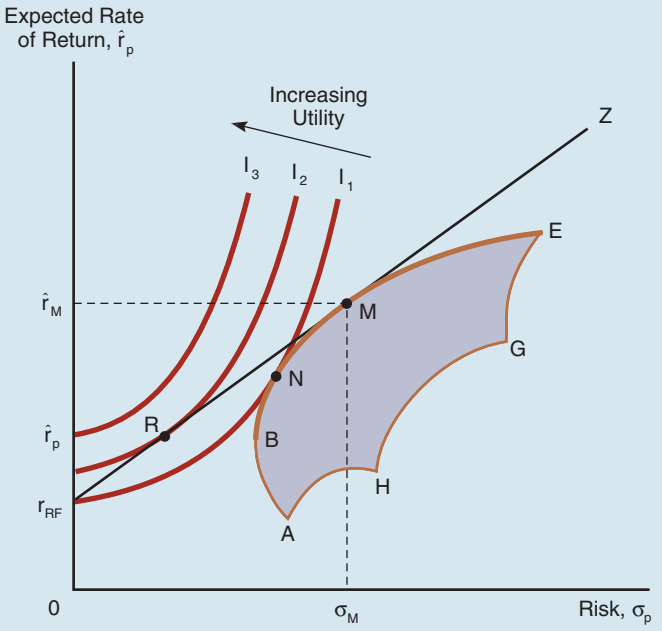
However, the investor can do better than Portfolio N; he or she can reach a higher indifference curve. In addition to the feasible set of risky portfolios, we now have a risk-free asset that provides a riskless return, r_{RF}. Given the risk-free asset, investors can create new portfolios that combine the risk-free asset with a

[4]The CAPM was originated by William F. Sharpe in his article "Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk," which appeared in the September 1964 issue of the *Journal of Finance*. Note that Professor Sharpe won the Nobel Prize in economics for his capital asset pricing work. The assumptions inherent in Sharpe's model were spelled out by Michael C. Jensen in "Capital Markets: Theory and Evidence," *Bell Journal of Economics and Management Science*, Autumn 1972, pp. 357–398.

[5]With no restrictions on short sales, an asset's weight in the portfolio may be negative or greater than 1 as long as the sum of all asset weights equals 1.

Figure 7-5

Investor Equilibrium: Combining the Risk-Free Asset with the Market Portfolio



portfolio of risky assets. This enables them to achieve any combination of risk and return on the straight line connecting r_{RF} with M, the point of tangency between that straight line and the efficient frontier of risky asset portfolios.[6] Some portfolios on the line $r_{RF}MZ$ will be preferred to most risky portfolios on the efficient frontier BNME, so the points on the line $r_{RF}MZ$ now represent the best attainable combinations of risk and return.

Given the new opportunities along line $r_{RF}MZ$, our investor will move from Point N to Point R, which is on his or her highest attainable risk/return

[6]The risk/return combinations between a risk-free asset and a risky asset (a single stock or a portfolio of stocks) will always be linear. To see this, consider the following equations, which were developed earlier, for return, $\hat{r}_p$, and risk, σ_p, for any combination w_{RF} and $(1 - w_{RF})$:

$$\hat{r}_p = w_{RF} r_{RF} + (1 - w_{RF}) \hat{r}_M \quad (7\text{-}1a)$$

and

$$\sigma_P = \sqrt{w_{RF}^2\sigma_{RF}^2 + (1 - w_{RF})^2\sigma_M^2 + 2w_{RF}(1 - w_{RF})\rho_{RF,M}\sigma_{RF}\sigma_M}. \quad (7\text{-}2a)$$

Equation 7-1a is linear. As for Equation 7-2a, we know that r_{RF} is the risk-free asset, so $\sigma_{RF} = 0$; hence, σ_{RF}^2 is also zero. Using this information, we can simplify Equation 7-2a as follows:

$$\sigma_P = \sqrt{(1 - w_{RF})^2\sigma_M^2} = (1 - w_{RF})\sigma_M. \quad (7\text{-}2b)$$

Thus, σ_p is also linear when a riskless asset is combined with a portfolio of risky assets.

If expected returns, as measured by $\hat{r}_p$, and risk, as measured by σ_p, are both linear functions of w_{RF}, then the relationship between $\hat{r}_p$ and σ_p, when graphed as in Figure 7-5, must also be linear. For example, if 100% of the portfolio is invested in r_{RF} with a return of 8%, the portfolio return will be 8% and σ_p will be 0. If 100% is invested in M, with $r_M = 12\%$ and $\sigma_M = 10\%$, then $\sigma_p = 1.0(10\%) = 10\%$, and $\hat{r}_p = 0(8\%) + 1.0(12\%) = 12\%$. If 50% of the portfolio is invested in M and 50% in the risk-free asset, then $\sigma_p = 0.5(10\%) = 5\%$, and $\hat{r}_p = 0.5(8\%) + 0.5(12\%) = 10\%$. Plotting these points will reveal the linear relationship given as $r_{RF}MZ$ in Figure 7-5.

indifference curve. Note that any point on the old efficient frontier BNME (except the point of tangency M) is dominated by some point along the line $r_{RF}MZ$. In general, since investors can include both the risk-free security and a fraction of the risky portfolio, M, in a portfolio, it will be possible to move to a point such as R. In addition, if the investor can borrow as well as lend (lending is equivalent to buying risk-free debt securities) at the riskless rate r_{RF}, it is possible to move out on the line segment MZ, and one would do so if his or her indifference curve were tangent to $r_{RF}MZ$ to the right of Point M.[7]

All investors should hold portfolios lying on the line $r_{RF}MZ$ under the conditions assumed in the CAPM. This implies that they should hold portfolios that are combinations of the risk-free security and the risky portfolio M. Thus, the addition of the risk-free asset totally changes the efficient set: The efficient set now lies along line $r_{RF}MZ$ rather than along the curve BNME. Also, note that if the capital market is to be in equilibrium, M must be a portfolio that contains every risky asset in exact proportion to that asset's fraction of the total market value of all assets; that is, if Security i is X percent of the total market value of all securities, X percent of the market portfolio M must consist of Security i. (In other words, M is the market-value-weighted portfolio of *all* risky assets in the economy.) Thus, all investors should hold portfolios that lie on the line $r_{RF}MZ$, with the particular location of a given individual's portfolio being determined by the point at which his or her indifference curve is tangent to the line.

The line $r_{RF}MZ$ in Figure 7-5 is called the **Capital Market Line (CML).** It has an intercept of r_{RF} and a slope of $(\hat{r}_M - r_{RF})/\sigma_M$.[8] Therefore, the equation for the Capital Market Line may be expressed as follows:

$$\text{CML: } \hat{r}_p = r_{RF} + \left(\frac{\hat{r}_M - r_{RF}}{\sigma_M}\right)\sigma_p. \tag{7-6}$$

The expected rate of return *on an efficient portfolio* is equal to the riskless rate plus a risk premium that is equal to $(\hat{r}_M - r_{RF})/\sigma_M$ multiplied by the portfolio's standard deviation, σ_p. Thus, the CML specifies a linear relationship between an efficient portfolio's expected return and risk, with the slope of the CML being equal to the expected return on the market portfolio of risky stocks, $\hat{r}_M$, minus the risk-free rate, r_{RF}, which is called the **market risk premium,** all divided by the standard deviation of returns on the market portfolio, σ_M:

$$\text{Slope of the CML} = (\hat{r}_M - r_{RF})/\sigma_M.$$

[7]An investor who is highly averse to risk will have a steep indifference curve and will end up holding only the riskless asset, or perhaps a portfolio at a point such as R, holding some of the risky market portfolio and some of the riskless asset. An investor only slightly averse to risk will have a relatively flat indifference curve, which will cause him or her to move out beyond M toward Z, borrowing to do so. This investor might buy stocks on margin, which means borrowing and using the stocks as collateral. If individuals' borrowing rates are higher than r_{RF}, then the line $r_{RF}MZ$ will tilt down (that is, be less steep) beyond M. This condition would invalidate the basic CAPM, or at least require it to be modified. Therefore, the assumption of being able to borrow or lend at the same rate is crucial to CAPM theory.

[8]Recall that the slope of any line is measured as $\Delta Y/\Delta X$, or the change in height associated with a given change in horizontal distance. r_{RF} is at 0 on the horizontal axis, so $\Delta X = \sigma_M - 0 = \sigma_M$. The vertical axis difference associated with a change from r_{RF} to $\hat{r}_M$ is $\hat{r}_M - r_{RF}$. Therefore, slope $= \Delta Y/\Delta X = (\hat{r}_M - r_{RF})/\sigma_M$.

For example, suppose $r_{RF} = 10\%$, $\hat{r}_M = 15\%$, and $\sigma_M = 15\%$. Then, the slope of the CML would be $(15\% - 10\%)/15\% = 0.33\%$, and if a particular efficient portfolio had $\sigma_p = 10\%$, then its $\hat{r}_p$ would be

$$\hat{r}_p = 10\% + 0.33(10\%) = 13.3\%.$$

A riskier portfolio with $\sigma_p = 20\%$ would have $\hat{r}_p = 10\% + 0.33(20\%) = 16.6\%$.

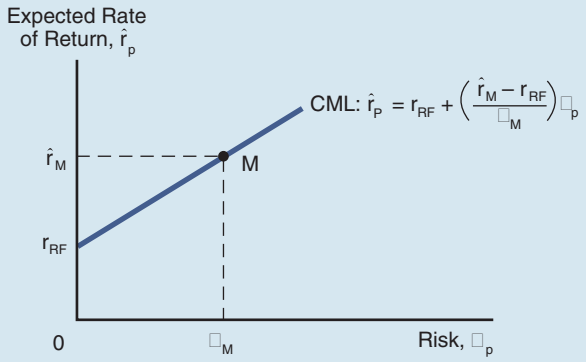
The CML is graphed in Figure 7-6. It is a straight line with an intercept at r_{RF} and a slope equal to the market risk premium $(r_M - r_{RF})$ divided by σ_M. The slope of the CML reflects the aggregate attitude of investors toward risk.

Note that an efficient portfolio is one that is well diversified; hence all of its unsystematic risk has been eliminated and its only remaining risk is market risk. Therefore, unlike individual stocks, the risk of an efficient portfolio is measured by its standard deviation, σ_p. The CML equation specifies the relationship between risk and return for such efficient portfolios, that is, for portfolios that lie on the CML, and in the CML equation and graph, risk is measured by portfolio standard deviation.

The CML specifies the relationship between risk and return for an efficient portfolio, but investors and managers are more concerned about the relationship between risk and return for *individual assets*. To develop the risk/return relationship for individual securities, note in Figure 7-5 that all investors are assumed to hold portfolio M, so M must be the market portfolio, that is, the one that contains all stocks. Note also that M is an *efficient* portfolio. Thus, the CML defines the relationship between the market portfolio's expected return and its standard deviation. Equations 7-4 and 7-5 show the formulas for the expected return and standard deviation for a multi-asset portfolio, including the market portfolio. It is possible to take the equations for the expected return and standard deviation of a multi-asset portfolio and show that the required return for each individual stock i

Figure 7-6

The Capital Market Line (CML)



Note: We did not draw it in, but you can visualize the shaded space shown in Figure 7-5 in this graph and the CML as the line formed by connecting r_{RF} with the tangent to the shaded space.

must conform to the following equation in order for the CML to hold for the market portfolio:[9]

$$\begin{aligned} r_i &= r_{RF} + \frac{(r_M - r_{RF})}{\sigma_M}\left(\frac{Cov(r_i, r_M)}{\sigma_M}\right) \\ &= r_{RF} + (r_M - r_{RF})\left(\frac{Cov(r_i, r_M)}{\sigma_M^2}\right). \end{aligned} \tag{7-7}$$

The CAPM defines the beta coefficient of company i, b_i, as follows:

$$\begin{aligned} b_i &= \frac{\text{Covariance between Stock i and the market}}{\text{Variance of market returns}} = \frac{Cov(r_i, r_M)}{\sigma_M^2} \\ &= \frac{\rho_{iM}\sigma_i\sigma_M}{\sigma_M^2} = \rho_{iM}\left(\frac{\sigma_i}{\sigma_M}\right). \end{aligned} \tag{7-8}$$

Recall that the risk premium for the market, RP_M, is $r_M - r_{RF}$. Using this definition and substituting Equation 7-8 into Equation 7-7 gives the **Security Market Line (SML):**

$$\begin{aligned} \text{SML: } r_i &= r_{RF} + (r_M - r_{RF})b_i \\ &= r_{RF} + (RP_M)b_i. \end{aligned} \tag{7-9}$$

The SML tells us that an individual stock's required return is equal to the risk-free rate plus a premium for bearing risk. The premium for risk is equal to the risk premium for the market, RP_M, multiplied by the risk of the individual stock, as measured by its beta coefficient. The beta coefficient measures the amount of risk that the stock contributes to the market portfolio.

Unlike the CML for a well-diversified portfolio, the SML tells us that the standard deviation (σ_i) of an individual stock should not be used to measure its risk, because some of the risk as reflected by σ_i can be eliminated by diversification. Beta reflects risk after taking diversification benefits into account, so beta, rather than σ_i, is used to measure individual stocks' risks to investors. Be sure to keep in mind the distinction between the SML and the CML, and why that distinction exists.

SELF-TEST

Draw a graph showing the feasible set of risky assets, the efficient frontier, the risk-free asset, and the CML.
Write out the equation for the CML and explain its meaning.
Write out the equation for the SML and explain its meaning.
What is the difference between the CML and the SML?
The standard deviation of stock returns of Park Corporation is 60%. The standard deviation of the market return is 20%. If the correlation between Park and the market is 0.40, what is Park's beta? (1.2)

[9]For consistency with most investments textbooks, we let $Cov(r_i, r_M)$ denote the covariance between the returns of assets i and M. Using the notation in Chapter 6, we would have denoted the covariance as COV_{iM}.

7.5 Calculating Beta Coefficients

Equation 7-7 defines beta, but recall from Chapter 6 that this equation for beta is also the formula for the slope coefficient in a regression of the stock return against the market return. Therefore, beta can be calculated by plotting the historical returns of a stock on the y-axis of a graph versus the historical returns of the market portfolio on the x-axis, and fitting the regression line. In his 1964 article that set forth the CAPM, Sharpe called this regression line the **characteristic line.** Thus, a stock's beta is the slope of its characteristic line. In Chapter 6 we used this approach to calculate the beta for General Electric. In this chapter, we perform a more detailed analysis of the calculation of beta for General Electric, and we also perform a similar analysis for a portfolio of stocks, Fidelity's Magellan Fund.

Calculating the Beta Coefficient for a Single Stock: General Electric

Table 7-2 shows a summary of the data used in this analysis; the full data set is in the file **FM12 Ch 07 Tool Kit.xls**. Table 7-2 shows the market returns (defined as the percentage price change of the S&P 500), the stock returns for GE, and the returns on the Magellan Fund (which is a well-diversified portfolio). The table also shows the risk-free rate, defined as the rate on a short-term (3-month) U.S. Treasury bill, which we will use later in this analysis.

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See ***FM12 Ch 07 Tool Kit.xls*** at the textbook's Web site.

As Table 7-2 shows, GE had an average annual return of 6.9% during this 4-year period (June 2002 through May 2006), while the market had an average annual return of 5.4%. As we noted before, it is usually unreasonable to think that the future expected return for a stock will equal its average historical return over a relatively short period, such as 4 years. However, we might well expect past volatility to be a reasonable estimate of future volatility, at least during the next couple of years. Note that the standard deviation for GE's return during this period was 19.1% versus 13.0% for the market. Thus, the market's volatility is less than that of GE. This is what we would expect, since the market is a well-diversified portfolio and thus much of its risk has been diversified away. The correlation between GE's stock returns and the market returns is about 0.49, which is a little higher than the correlation for a typical stock.

Table 7-2 Summary of Data for Calculating Beta (June 2002–May 2006)

	r_M, Market Return (S&P 500 Index)	r_i, GE Return	r_p, Fidelity Megellan Fund Return	r_{RF}, Risk-Free Rate (Monthly Return on 3-Month T-Bill)
Average return (annual)	5.4%	6.9%	5.7%	2.1%
Standard deviation (annual)	13.0%	19.1%	13.5%	0.4%
Correlation with market return, ρ		0.49	0.98	−0.04

Figure 7-7

Calculating a Beta Coefficient for General Electric

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See ***FM12 Ch 07 Tool Kit.xls*** at the textbook's Web site for all calculations.

Figure 7-7 shows a plot of GE's returns against the market's returns. As you will notice if you look in the file ***FM12 Ch 07 Tool Kit.xls,*** we used the *Excel* chart feature to add a trend line and to display the equation and R^2 value on the chart itself. We also used the *Excel* regression analysis feature, which provides additional data.

Table 7-3 reports some of the regression results for GE. Its estimated beta, which is the slope coefficient, is about 0.72. This means that GE's beta is less than the average beta of 1.0. Therefore, GE moves up and down, on average, by less than the same percent as the market. As with all regression results, 0.72 is just an estimate of beta, and not necessarily the true value of beta. Table 7-3 also shows the t statistic and the probability that the true beta is zero. For GE, this probability is approximately equal to zero. This means that there is virtually a zero chance that the true beta is equal to zero. Since this probability is less than 5%, statisticians would say that the slope coefficient, beta, is "statistically significant." The output of the regression analysis also gives us the 95% confidence interval for the estimate of beta. For GE, the results tell us that we can be 95% confident that the true beta is between 0.35 and 1.10. This is an extremely wide range, but it is typical for most individual stocks. Therefore, the regression estimate for the beta of any single company is highly uncertain.

Note also that the points in Figure 7-7 are not clustered very tightly around the regression line. Sometimes GE does much better than the market; other times it does much worse. The R^2 value shown in the chart measures the degree of dispersion about the regression line. Statistically speaking, it measures the percent of variance that is explained by the regression equation. An R^2 of 1.0 indicates that all points lie exactly on the line, hence that all of the variance of the y-variable is explained by the x-variable. The R^2 for GE is about 0.24, which is typical for most individual stocks. This indicates that about 24% of the variance in GE's returns is explained by the market return.

Table 7-3

Regression Results for Calculating Beta

	Regression Coefficient	t Statistic	Probability of t Statistic	Lower 95% Confidence Interval	Upper 95% Confidence Interval
Panel a: General Electric (Market model)					
Intercept	0.00	0.35	0.73	−0.01	0.02
Slope	0.72	3.85	0.00	0.35	1.10
Panel b: Magellan Fund (Market model)					
Intercept	0.00	0.16	0.88	0.00	0.00
Slope	1.01	31.86	0.00	0.95	1.08
Panel c: General Electric (CAPM: Excess returns)					
Intercept	0.00	0.29	0.78	−0.01	0.02
Slope	0.73	3.87	0.00	0.35	1.11

Note: The market model uses actual historical returns; the CAPM model uses returns in excess of the risk-free rate.

Finally, note that the intercept shown in the regression equation displayed on the chart is about 0.0025. Since the regression equation is based on monthly data, this means that GE had a 3% average monthly return that was not explained by the CAPM model. However, the regression results in Table 7-3 also show that the probability of the t statistic is greater than 5%, meaning that the "true" intercept might be zero. Therefore, most statisticians would say that this intercept is not statistically significant—the returns of GE are so volatile that we cannot be sure that the true intercept is not equal to zero. Translating statistician-talk into English, this means that the part of GE's average monthly return that is not explained by the CAPM could very well be zero. Thus, the CAPM might very well explain all of GE's average monthly returns.

The Market Model versus the CAPM

Note that when we estimated beta, we used the following regression equation:

$$\bar{r}_{i,t} = a_i + b_i\bar{r}_{M,t} + e_{i,t} \quad (7\text{-}10)$$

where

$\bar{r}_{i,t}$ = historical (realized) rate of return on Stock i in period t.
$\bar{r}_{M,t}$ = historical (realized) rate of return on the market in period t.
a_i = vertical axis intercept term for Stock i.
b_i = slope, or beta coefficient, for Stock i.
$e_{i,t}$ = random error, reflecting the difference between the actual return on Stock i in a given period and the return as predicted by the regression line.

Equation 7-10 is called the **market model,** because it regresses the stock's return against the market's return. However, the SML of the CAPM for realized returns is a little different from Equation 7-10, as is shown below:

$$\text{SML for realized returns: } \bar{r}_{i,t} = \bar{r}_{RF,t} + b_i(\bar{r}_{M,t} - \bar{r}_{RF,t}) + e_{i,t}, \quad \textbf{(7-11)}$$

where $\bar{r}_{RF,t}$ is the historical (realized) risk-free rate in period t.

To use the CAPM to estimate beta, we must rewrite Equation 7-11 to be a regression equation by adding an intercept, a_i. The resulting regression equation is

$$(\bar{r}_i - \bar{r}_{RF,t}) = a_i + b_i(\bar{r}_{M,t} - \bar{r}_{RF,t}) + e_{i,t}. \quad \textbf{(7-12)}$$

Therefore, to be theoretically correct when estimating beta, we should use the stock's return in excess of the risk-free rate as the y-variable and the market's return in excess of the risk-free rate as the x-variable. We did this for GE using the data in Table 7-2, and the results are reported in Panel c of Table 7-3. Note that there are no appreciable differences between the results in Panel a, the market model, and in Panel c, the CAPM model. This typically is the case, so we will use the market model in the rest of this book.

Calculating the Beta Coefficient for a Portfolio: The Magellan Fund

Let's calculate beta for the Magellan Fund, which is a well-diversified portfolio. Figure 7-8 shows the plot of Magellan's monthly returns versus the market's monthly returns. Note the differences between this chart and the one for GE shown in Figure 7-7. The points for Magellan are tightly clustered around the regression line, indicating that the vast majority of Magellan's volatility is explained by the stock market. The R^2 of over 0.95 confirms this visual conclusion. We can also see from Table 7-2 that the Magellan Fund has a standard deviation of 13.5%, which is only slightly higher than the 13.0% standard deviation of the market.

As Table 7-3 shows, the estimated beta is 1.01, and the 95% confidence interval is from 0.95 to 1.08, which is much tighter than the one for GE. The intercept is virtually zero, and the probability of the intercept's t statistic is greater than 5%. Therefore, the intercept is statistically insignificant, indicating that the CAPM explains the average monthly return of the Magellan Fund very well.

Mutual fund managers are often evaluated by their risk-adjusted performance. The three most widely used measures are *Jensen's alpha, Sharpe's reward-to-variability ratio,* and *Treynor's reward-to-volatility ratio.* **Jensen's alpha,** which is the intercept in a CAPM regression of excess returns, is 0.25% per year for Magellan, which seems to indicate that the Magellan fund had slightly superior performance. However, this intercept was not statistically significantly different from zero. Its t statistic is 0.18, which is so low a value that it could happen about 86% of the time by chance even if the intercept is truly zero. If this probability is greater than 5%, as is the case for Magellan, most statisticians will be reluctant to conclude that Magellan's excess return of 0.25% is truly better than zero, not just a result of pure chance.

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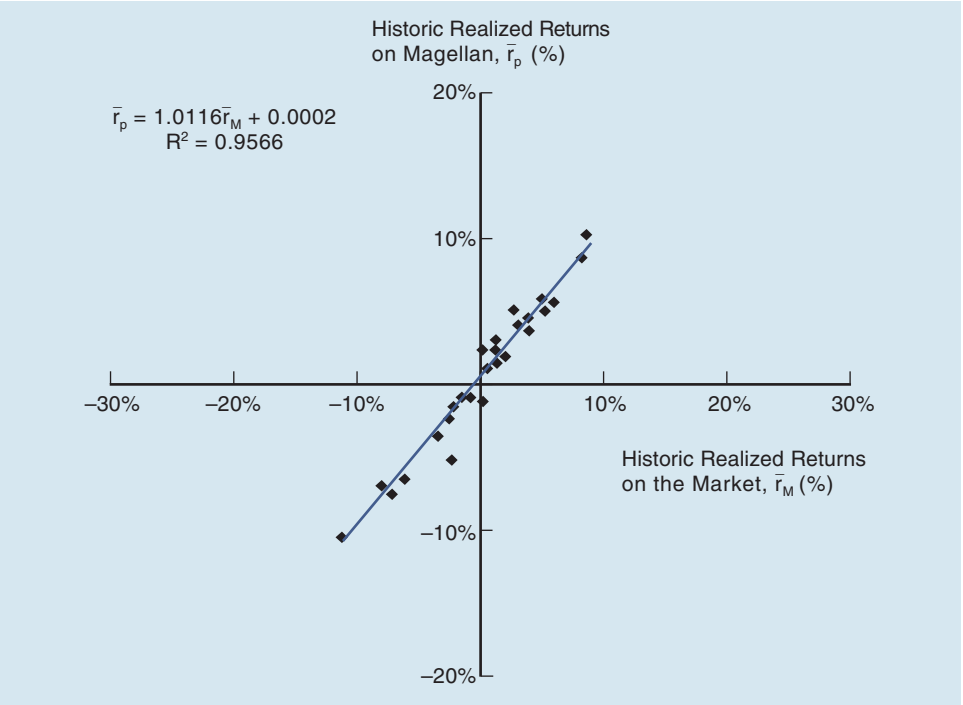
See ***FM12 Ch 07 Tool Kit.xls*** at the textbook's Web site for all calculations for these measures.

Figure 7-8

Calculating a Beta Coefficient for Fidelity's Magellan Fund



Sharpe's reward-to-variability ratio is defined as the portfolio's average return (in excess of the risk-free rate) divided by its standard deviation. Sharpe's ratio for Magellan during the past 4 years is 0.27, which is almost the same as the S&P's measure of 0.26.

Treynor's reward-to-volatility ratio is defined as the portfolio's average return (in excess of the risk-free rate) divided by its beta. For Magellan, this is 3.6%, which is a little better than the S&P 500's ratio 3.3%. All in all, the Magellan fund seems to have slightly outperformed the market, but perhaps not by a statistically significant amount. While it is not clear whether or not Magellan "beat the market," it did dramatically reduce the risk faced by investors vis-à-vis the risk inherent in a randomly chosen individual stock.

Additional Insights into Risk and Return

The CAPM provides some additional insights into the relationship between risk and return.

1. The relationship between a stock's total risk, market risk, and diversifiable risk can be expressed as follows:

$$\text{Total risk} = \text{Variance} = \text{Market risk} + \text{Diversifiable risk} \qquad (7\text{-}13)$$

$$\sigma_i^2 = b_i^2\sigma_M^2 + \sigma_{e_i}^2.$$

Here σ_i^2 is the variance (or total risk) of Stock i, σ_M^2 is the variance of the market, b_i is Stock i's beta coefficient, and $\sigma_{e_i}^2$ is the variance of Stock i's regression error term.

2. If in Figure 7-7 all the points had plotted exactly on the regression line, then the variance of the error term, $\sigma_{e_i}^2$, would have been zero, and all of the stock's total risk would have been market risk. On the other hand, if the points were widely scattered about the regression line, much of the stock's total risk would be diversifiable. The shares of a large, well-diversified mutual fund will plot very close to the regression line.
3. Beta is a measure of relative market risk, but the *actual* market risk of Stock i is $b_i^2\sigma_M^2$. Market risk can also be expressed in standard deviation form, $b_i\sigma_M$. The higher a stock's beta, the higher its market risk. If beta were zero, the stock would have no market risk, while if beta were 1.0, the stock would be exactly as risky as the market—assuming the stock is held in a diversified portfolio—and the stock's market risk would be σ_M.

Advanced Issues in Calculating Beta

Betas are generally estimated from the stock's characteristic line by running a linear regression between past returns on the stock in question and past returns on some market index. We define betas developed in this manner as **historical betas.** However, in most situations, it is the *future* beta that is needed. This has led to the development of two different types of betas: (1) adjusted betas and (2) fundamental betas.

Adjusted betas grew largely out of the work of Marshall E. Blume, who showed that true betas tend to move toward 1.0 over time.[10] Therefore, we can begin with a firm's pure historical statistical beta, make an adjustment for the expected future movement toward 1.0, and produce an adjusted beta that will, on average, be a better predictor of the future beta than would the unadjusted historical beta. *Value Line* publishes betas based on approximately this formula:

$$\text{Adjusted beta} = 0.67(\text{Historical beta}) + 0.35(1.0).$$

Consider American Camping Corporation, a retailer of supplies for outdoor activities. ACC's historical beta is 1.2. Therefore, its adjusted beta is

$$\text{Adjusted beta} = 0.67(1.2) + 0.35(1.0) = 1.15.$$

Other researchers have extended the adjustment process to include such fundamental risk variables as financial leverage, sales volatility, and the like. The end product here is a **fundamental beta.**[11] These betas are constantly adjusted to reflect changes in a firm's operations and capital structure, whereas with historical betas (including adjusted ones), such changes might not be reflected until several years after the company's "true" beta had changed.

Adjusted betas are obviously heavily dependent on unadjusted historical betas, and so are fundamental betas as they are actually calculated. Therefore, the plain old historical beta, calculated as the slope of the characteristic line, is important even if

[10]See Marshall E. Blume, "Betas and Their Regression Tendencies," *Journal of Finance*, June 1975, pp. 785–796, and Marshall E. Blume, "On the Assessment of Risk," *Journal of Finance*, March 1971, pp. 1–10.
[11]See Barr Rosenberg and James Guy, "Beta and Investment Fundamentals," *Financial Analysts Journal*, May–June 1976, pp. 60–72. Rosenberg, then a professor at the University of California at Berkeley, later founded a company that calculates fundamental betas by a proprietary procedure and then sells them to institutional investors.

one goes on to develop a more exotic version. With this in mind, it should be noted that several different sets of data can be used to calculate historical betas, and the different data sets produce different results. Here are some points to note:

1. Betas can be based on historical periods of different lengths. For example, data for the past 1, 2, 3, and so on, years may be used. Most people who calculate betas today use 5 years of data, but this choice is arbitrary, and different lengths of time usually alter significantly the calculated beta for a given company.
2. Returns may be calculated over holding periods of different lengths—a day, a week, a month, a quarter, a year, and so on. For example, if it has been decided to analyze data on NYSE stocks over a 5-year period, then we might obtain 52(5) = 260 weekly returns on each stock and on the market index. We could also use 12(5) = 60 monthly returns, or 1(5) = 5 annual returns. The set of returns on each stock, however large the set turns out to be, would then be regressed on the corresponding market returns to obtain the stock's beta. In statistical analysis, it is generally better to have more rather than fewer observations, because using more observations generally leads to greater statistical confidence. This suggests the use of weekly returns, and, say, 5 years of data, for a sample size of 260, or even daily returns for a still larger sample size. However, the shorter the holding period, the more likely the data are to exhibit random "noise." Also, the greater the number of years of data, the more likely it is that the company's basic risk position has changed. Thus, the choice of both the number of years of data and the length of the holding period for calculating rates of return involves trade-offs between a desire to have many observations versus a desire to rely on recent and consequently more relevant data.
3. The value used to represent "the market" is also an important consideration, as the index used can have a significant effect on the calculated beta. Many analysts today use the New York Stock Exchange Composite Index (based on more than 2,000 common stocks, weighted by the value of each company), but others use the S&P 500 Index. In theory, the broader the index, the better the beta. Indeed, the index should really include returns on all stocks, bonds, leases, private businesses, real estate, and even "human capital." As a practical matter, however, we cannot get accurate returns data on most other types of assets, so measurement problems largely restrict us to stock indexes.

Where does this leave financial managers regarding the proper beta? They must "pay their money and take their choice." Some managers calculate their own betas, using whichever procedure seems most appropriate under the circumstances. Others use betas calculated by organizations such as Yahoo!Finance or *Value Line,* perhaps using one service or perhaps averaging the betas of several services. The choice is a matter of judgment and data availability, for there is no "right" beta. Generally, though, the betas derived from different sources will, for a given company, be reasonably close together. If they are not, then our confidence in using the CAPM will be diminished.

SELF-TEST

Explain the meaning and significance of a stock's beta coefficient. Illustrate your explanation by drawing, on one graph, the characteristic lines for stocks with low, average, and high risk. (Hint: Let your three characteristic lines intersect at $\bar{r}_i = \bar{r}_M = 6\%$, the assumed risk-free rate.)
What is a typical R^2 for the characteristic line of an individual stock? For a portfolio?
What is the market model? How is it different from the SML for the CAPM?
How are stand-alone risk, market risk, and diversifiable risk related?

7.6 Empirical Tests of the CAPM

Does the CAPM's SML produce reasonable estimates for a stock's required return? The literature dealing with empirical tests of the CAPM is quite extensive, so we can give here only a synopsis of some of the key work.

Tests of the Stability of Beta Coefficients

According to the CAPM, the beta used to estimate a stock's market risk should reflect investors' estimates of the stock's *future* volatility in relation to that of the market. Obviously, we do not know now how a stock will be related to the market in the future, nor do we know how the average investor views this expected future relative volatility. All we have are data on past volatility, which we can use to plot the characteristic line and to calculate *historical betas*. If historical betas have been stable over time, then there would seem to be reason for investors to use past betas as estimators of future volatility. For example, if Stock i's beta had been stable in the past, then its historical b_i would probably be a good proxy for its *ex ante*, or expected, beta. By "stable" we mean that if b_i were calculated with data from the period of, say, 2002 to 2006, then this same beta (approximately) should be found from 2007 to 2011.

Robert Levy, Marshall Blume, and others have studied the question of beta stability in depth.[12] Levy calculated betas for individual securities, as well as for portfolios of securities, over a range of time intervals. He concluded (1) that the betas of individual stocks are unstable, hence that past betas for *individual securities* are *not* good estimators of their future risk, but (2) that betas of portfolios of 10 or more randomly selected stocks are reasonably stable, hence that past *portfolio* betas are good estimators of future portfolio volatility. In effect, the errors in individual securities' betas tend to offset one another in a portfolio. The work of Blume and others supports this position.

The conclusion that follows from the beta stability studies is that the CAPM is a better concept for structuring investment portfolios than it is for estimating the required return for individual securities.[13]

Tests of the CAPM Based on the Slope of the SML

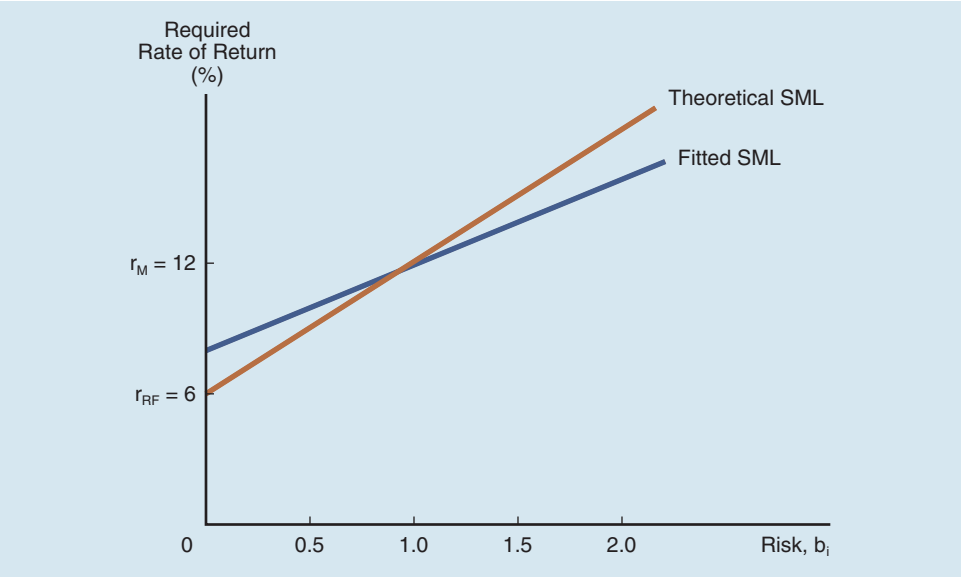
The CAPM states that a linear relationship exists between a security's required rate of return and its beta. Further, when the SML is graphed, the vertical axis intercept should be r_{RF}, and the required rate of return for a stock (or portfolio) with $b = 1.0$ should be r_M, the required rate of return on the market. Various researchers have attempted to test the validity of the CAPM by calculating betas and realized rates of return, plotting these values in graphs such as that in Figure 7-9, and then observing whether or not (1) the intercept is equal to r_{RF}, (2) the plot is linear, and (3) the line passes through the point $b = 1.0$, r_M. Monthly or daily historical rates of return are generally used for stocks, and both 30-day Treasury bill rates and long-term Treasury bond rates have been used to estimate the value

[12]See Robert A. Levy, "On the Short-Term Stationarity of Beta Coefficients," *Financial Analysts Journal*, November—December 1971, pp. 55–62; and Marshall E. Blume, "Betas and Their Regression Tendencies," *Journal of Finance*, June 1975, pp. 785–796.

[13]For more on beta stability, see Robert W. Kolb and Ricardo J. Rodriguez, "The Regression Tendencies of Betas: A Reappraisal," *The Financial Review*, May 1989, pp. 319–334. Also see Robert Kolb, "Is the Distribution of Betas Stationary?" *Journal of Financial Research*, Winter 1990, pp. 279–283.

Figure 7-9

Tests of the CAPM



of r_{RF}. Also, most of the studies actually analyzed portfolios rather than individual securities because security betas are so unstable.

Before discussing the results of the tests, it is critical to recognize that although the CAPM is an ex ante, or forward-looking, model, the data used to test it are entirely historical. This presents a problem, for there is no reason to believe that *realized* rates of return over past holding periods are necessarily equal to the rates of return people *expect* in the future. Also, historical betas may or may not reflect expected future risk. This lack of ex ante data makes it extremely difficult to test the CAPM, but for what it is worth, here is a summary of the key results:

1. The evidence generally shows a significant positive relationship between realized returns and beta. However, the slope of the relationship is usually less than that predicted by the CAPM.
2. The relationship between risk and return appears to be linear. Empirical studies give no evidence of significant curvature in the risk/return relationship.
3. Tests that attempt to assess the relative importance of market and company-specific risk do not yield conclusive results. The CAPM implies that company-specific risk should not be relevant, yet both kinds of risk appear to be positively related to security returns; that is, higher returns seem to be required to compensate for diversifiable as well as market risk. However, it may be that the observed relationships reflect statistical problems rather than the true nature of capital markets.
4. Richard Roll has questioned whether it is even conceptually possible to test the CAPM.[14] Roll showed that the linear relationship that prior researchers had observed in graphs like Figure 7-9 resulted from the mathematical properties of the models being tested, hence that a finding of linearity proved nothing

[14]See Richard Roll, "A Critique of the Asset Pricing Theory's Tests," *Journal of Financial Economics*, March 1977, pp. 129–176.

whatsoever about the validity of the CAPM. Roll's work did not disprove the CAPM, but he did show that it is virtually impossible to prove that investors behave in accordance with its predictions.

5. If the CAPM were completely valid, it should apply to all financial assets, including bonds. In fact, when bonds are introduced into the analysis, they *do not* plot on the SML. This is worrisome, to say the least.

Current Status of the CAPM

The CAPM is extremely appealing at an intellectual level: It is logical and rational, and once someone works through and understands the theory, his or her reaction is usually to accept it without question. However, doubts begin to arise when one thinks about the assumptions upon which the model is based, and these doubts are as much reinforced as reduced by the empirical tests. Our own views as to the current status of the CAPM are as follows:

1. The CAPM framework, with its focus on market as opposed to stand-alone risk, is clearly a useful way to think about the risk of assets. Thus, as a conceptual model, the CAPM is of truly fundamental importance.
2. When applied in practice, the CAPM appears to provide neat, precise answers to important questions about risk and required rates of return. However, the answers are less clear than they seem. The simple truth is that we do not know precisely how to measure any of the inputs required to implement the CAPM. These inputs should all be ex ante, yet only ex post data are available. Further, historical data on $\bar{r}_M$, r_{RF}, and betas vary greatly depending on the time period studied and the methods used to estimate them. Thus, although the CAPM appears precise, estimates of r_i found through its use are subject to potentially large errors.[15]
3. Because the CAPM is logical in the sense that it represents the way risk-averse people ought to behave, the model is a useful conceptual tool.
4. It is appropriate to think about many financial problems in a CAPM framework. However, it is important to recognize the limitations of the CAPM when using it in practice.

SELF-TEST

What are the two major types of tests that have been performed to test the validity of the CAPM? Explain their results. (Beta stability and slope of the SML.)
Are there any reasons to question the validity of the CAPM? Explain.

7.7 Arbitrage Pricing Theory

The CAPM is a single-factor model. That is, it specifies risk as a function of only one factor, the security's beta coefficient. Perhaps the risk/return relationship is more complex, with a stock's required return a function of more than one factor. For example, what if investors, because personal tax rates on capital gains are lower than those on dividends, value capital gains more highly than dividends? Then, if two stocks had the same market risk, the stock paying the higher dividend would have the higher required rate of return. In that case, required returns would be a function of two factors, market risk and dividend policy.

[15]For an article supporting a positive link between market risk and expected return, see Felicia Marston and Robert S. Harris, "Risk and Return: A Revisit Using Expected Returns," *Financial Review*, February 1993, pp. 117–137.

Further, what if many factors are required to specify the equilibrium risk/return relationship rather than just one or two? Stephen Ross has proposed an approach called the **Arbitrage Pricing Theory (APT).**[16] The APT can include any number of risk factors, so the required return could be a function of two, three, four, or more factors. We should note at the outset that the APT is based on complex mathematical and statistical theory that goes far beyond the scope of this text. Also, although the APT model is widely discussed in academic literature, practical usage to date has been limited. However, usage may increase, so students should at least have an intuitive idea of what the APT is all about.

The SML states that each stock's required return is equal to the risk-free rate plus the product of the market risk premium times the stock's beta coefficient. Assuming stocks are in equilibrium, the required return will be equal to the expected return:

$$\hat{r}_i = r_i = r_{RF} + (r_M - r_{RF})b_i .$$

The historical realized return, $\bar{r}_i$, which will generally be different from the expected return, can be expressed as follows:[17]

$$\bar{r}_i = \hat{r}_i + (\bar{r}_M - \hat{r}_M)b_i + e_i. \quad \textbf{(7-14)}$$

Thus, the realized return, $\bar{r}_i$, will be equal to the expected return, $\hat{r}_i$, plus a positive or negative increment, $(\bar{r}_M - \hat{r}_M)b_i$, which depends jointly on the stock's beta and whether the market did better or worse than was expected, plus a random error term, e_i.

The market's realized return, $\bar{r}_M$, is in turn determined by a number of factors, including domestic economic activity as measured by gross domestic product (GDP), the strength of the world economy, the level of inflation, changes in tax laws, and so forth. Further, different groups of stocks are affected in different ways by these fundamental factors. So, rather than specifying a stock's return as a function of one factor (return on the market), one could specify required and realized returns on individual stocks as a function of various fundamental economic factors. If this were done, we would transform Equation 7-14 into 7-15:

$$\bar{r}_i = \hat{r}_i + (\bar{F}_1 - \hat{F}_1)b_{i1} + \cdots + (\bar{F}_j - \hat{F}_j)b_{ij} + e_i. \quad \textbf{(7-15)}$$

Here,

$\bar{r}_i$ = realized rate of return on Stock i.
$\hat{r}_i$ = expected rate of return on Stock i.
$\bar{F}_j$ = realized value of economic Factor j.
$\hat{F}_j$ = expected value of Factor j.
b_{ij} = sensitivity of Stock i to economic Factor j.
e_i = effect of unique events on the realized return of Stock i.

[16]See Stephen A. Ross, "The Arbitrage Theory of Capital Asset Pricing," *Journal of Economic Theory*, December 1976, pp. 341–360.
[17]To avoid cluttering the notation, we have dropped the subscript t to denote a particular time period.

Equation 7-15 shows that the realized return on any stock is equal to (1) the stock's expected return, (2) increases or decreases that depend on unexpected changes in fundamental economic factors times the sensitivity of the stock to these changes, and (3) a random term that reflects changes unique to the firm.

Certain stocks or groups of stocks are most sensitive to Factor 1, others to Factor 2, and so forth, and every portfolio's returns depend on what happened to the different fundamental factors. Theoretically, one could construct a portfolio such that (1) the portfolio was riskless and (2) the net investment in it was zero (some stocks would be sold short, with the proceeds from the short sales being used to buy the stocks held long). Such a zero investment portfolio must have a zero expected return, or else arbitrage operations would occur and cause the prices of the underlying assets to change until the portfolio's expected return was zero. Using some complex mathematics and a set of assumptions including the possibility of short sales, the APT equivalent of the CAPM's Security Market Line can be developed from Equation 7-15:[18]

$$r_i = r_{RF} + (r_1 - r_{RF})b_{i1} + \cdots + (r_j - r_{RF})b_{ij}. \qquad \textbf{(7-16)}$$

Here r_j is the required rate of return on a portfolio that is sensitive only to the *j*th economic factor ($b_{pj} = 1.0$) and has zero sensitivity to all other factors. Thus, for example, $(r_2 - r_{RF})$ is the risk premium on a portfolio with $b_{p2} = 1.0$ and all other $b_{pj} = 0.0$. Note that Equation 7-16 is identical in form to the SML, but it permits a stock's required return to be a function of multiple factors.

To illustrate the APT concept, assume that all stocks' returns depend on only three risk factors: inflation, industrial production, and the aggregate degree of risk aversion (the cost of bearing risk, which we assume is reflected in the spread between the yields on Treasury and low-grade bonds). Further, suppose (1) the risk-free rate is 8.0%; (2) the required rate of return is 13% on a portfolio with unit sensitivity (b = 1.0) to inflation and zero sensitivities (b = 0.0) to industrial production and degree of risk aversion; (3) the required return is 10% on a portfolio with unit sensitivity to industrial production and zero sensitivities to inflation and degree of risk aversion; and (4) the required return is 6% on a portfolio (the risk-bearing portfolio) with unit sensitivity to the degree of risk aversion and zero sensitivities to inflation and industrial production. Finally, assume that Stock i has factor sensitivities (betas) of 0.9 to the inflation portfolio, 1.2 to the industrial production portfolio, and −0.7 to the risk-bearing portfolio. Stock i's required rate of return, according to the APT, would be 16.3%:

$$\begin{aligned} r_i &= 8\% + (13\% - 8\%)0.9 + (10\% - 8\%)1.2 + (6\% - 8\%)(-0.7) \\ &= 16.3\%. \end{aligned}$$

Note that if the required rate of return on the market were 15.0% and Stock i had a CAPM beta of 1.1, then its required rate of return, according to the SML, would be 15.7%:

$$r_i = 8\% + (15\% - 8\%)1.1 = 15.7\%.$$

[18]See Thomas E. Copeland, J. Fred Weston, and Kuldeep Shastri, *Financial Theory and Corporate Policy*, 4th Edition (Reading, MA: Addison-Wesley, 2005).

The primary theoretical advantage of the APT is that it permits several economic factors to influence individual stock returns, whereas the CAPM assumes that the effect of all factors, except those unique to the firm, can be captured in a single measure, the volatility of the stock with respect to the market portfolio. Also, the APT requires fewer assumptions than the CAPM and hence is more general. Finally, the APT does not assume that all investors hold the market portfolio, which is a CAPM requirement that clearly is not met in practice.

However, the APT faces several major hurdles in implementation, the most severe being that the APT does not identify the relevant factors. Thus, the APT does not tell us what factors influence returns, nor does it even indicate how many factors should appear in the model. There is some empirical evidence that only three or four factors are relevant: perhaps inflation, industrial production, the spread between low- and high-grade bonds, and the term structure of interest rates, but no one knows for sure.

The APT's proponents argue that it is not actually necessary to identify the relevant factors. Researchers use a statistical procedure called **factor analysis** to develop the APT parameters. Basically, they start with hundreds, or even thousands, of stocks and then create several different portfolios, where the returns on each portfolio are not highly correlated with returns on the other portfolios. Thus, each portfolio is apparently more heavily influenced by one of the unknown factors than are the other portfolios. Then, the required rate of return on each portfolio becomes the estimate for that unknown economic factor, shown as r_j in Equation 7-16. The sensitivities of each individual stock's returns to the returns on that portfolio are the factor sensitivities (betas). Unfortunately, the results of factor analysis are not easily interpreted; hence it does not provide significant insights into the underlying economic determinants of risk.[19]

SELF-TEST

What is the primary difference between the APT and the CAPM?
What are some disadvantages of the APT?
An analyst has modeled the stock of Brown Kitchen Supplies using a two-factor APT model. The risk-free rate is 5%, the required return on the first factor (r_1) is 10%, and the required return on the second factor (r_2) is 15%. If $b_{i1} = 0.5$ and $b_{i2} = 1.3$, what is Brown's required return? (20.5%)

7.8 The Fama-French Three-Factor Model

As we mentioned in Chapter 6, the results of two studies by Eugene F. Fama and Kenneth R. French of the University of Chicago seriously challenge the CAPM.[20] In the first of these studies, published in 1992, Fama and French hypothesized that the SML should have three factors. The first is the stock's CAPM beta, which measures the market risk of the stock. The second is the size of the company, measured by the market value of its equity (MVE), because if small companies are riskier than large companies, then we might expect small companies to have higher stock returns than large companies. The third factor is the book value of equity

[19]For additional discussion of the APT, see Eward L. Bubnys, "Simulating and Forecasting Utility Stock Returns: Arbitrage Pricing Theory vs. Capital Asset Pricing Model," *The Financial Review*, February 1990, pp. 1–23; David H. Goldenberg and Ashok J. Robin, "The Arbitrage Pricing Theory and Cost-of-Capital Estimation: The Case of Electric Utilities," *Journal of Financial Research*, Fall 1991, pp. 181–196; and Ashok Robin and Ravi Shukla, "The Magnitude of Pricing Errors in the Arbitrage Pricing Theory," *Journal of Financial Research*, Spring 1991, pp. 65–82.

[20]See Eugene F. Fama and Kenneth R. French, "The Cross-Section of Expected Stock Returns," *Journal of Finance*, 1992, pp. 427–465. Also see Eugene F. Fama and Kenneth R. French, "Common Risk Factors in the Returns on Stocks and Bonds," *Journal of Financial Economics*, 1993, pp. 3–56.

divided by the market value of equity, or the book-to-market ratio (B/M). If the market value is larger than the book value, then investors are optimistic about the stock's future. On the other hand, if the book value is larger than the market value, then investors are pessimistic about the stock's future, and it is likely that a ratio analysis will reveal that the company is experiencing sub-par operating performance and possibly even financial distress. In other words, a stock with a high B/M ratio might be risky, in which case investors would require a higher expected return to induce them to invest in such a stock.

When Fama and French tested their hypotheses, they found that small companies and companies with high B/M ratios had higher rates of return than the average stock, just as they hypothesized. Somewhat surprisingly, however, they found no relation between beta and return. After taking into account the returns due to the company's size and B/M ratio, high-beta stocks did not have higher-than-average returns, and low-beta stocks did not have lower-than-average returns.

In the second of their two studies, published in 1993, Fama and French developed a three-factor model based on their previous results. The first factor in the **Fama-French three-factor model** is the market risk premium, which is the market return, $\bar{r}_M$, minus the risk-free rate, $\bar{r}_{RF}$. Thus, their model begins like the CAPM, but they go on to add a second and third factor.[21] To form the second factor, they ranked all actively traded stocks by size and then divided them into two portfolios, consisting of small and big stocks. They calculated the return on each of these two portfolios, and created a third portfolio by subtracting the return on the big portfolio from that of the small one. They called this the SMB portfolio (for small size minus big size). This portfolio is designed to measure the variation in stock returns that is caused by the size effect.

To form the third factor, they ranked all stocks according to their book-to-market ratios (B/M). They placed the 30% of stocks with the highest ratios into a portfolio that they called the H portfolio (for high B/M ratios). They placed the 30% of stocks with the lowest ratios into a portfolio called the L portfolio (for low B/M ratios). They subtracted the return of the L portfolio from the H portfolio, and they called the result the HML portfolio (for high-B/M ratio minus low-B/M ratio). Their resulting model is shown here:

$$(\bar{r}_i - \bar{r}_{RF}) = a_i + b_i(\bar{r}_M - \bar{r}_{RF}) + c_i(\bar{r}_{SMB}) + d_i(\bar{r}_{HML}) + e_i, \qquad \textbf{(7-17)}$$

where

$\bar{r}_i$ = historical (realized) rate of return on Stock i.
$\bar{r}_{RF}$ = historical (realized) rate of return on the risk-free rate.
$\bar{r}_M$ = historical (realized) rate of return on the market.
$\bar{r}_{SMB}$ = historical (realized) rate of return on the small-size portfolio minus the big-size portfolio.
$\bar{r}_{HML}$ = historical (realized) rate of return on the high-B/M portfolio minus the low-B/M portfolio.
a_i = vertical axis intercept term for Stock i.
b_i, c_i, and d_i = slope coefficients for Stock i.
e_i = random error, reflecting the difference between the actual return on Stock i in a given period and the return as predicted by the regression line.

[21] Although our description captures the essence of their process for forming factors, their actual process is a little more complicated. The interested reader should see their 1993 paper as referenced in Footnote 20.

The Fama-French three-factor model version of the CAPM Security Market Line for the required return on a stock is

$$r_i = r_{RF} + a_i + b_i(r_M - r_{RF}) + c_i(r_{SMB}) + d_i(r_{HML}), \quad \textbf{(7-18)}$$

where $r_M - r_{RF}$ is the market risk premium, r_{SMB} is the expected value (i.e., premium) for the size factor, and r_{HML} is the expected value (i.e., premium) for the book/market factor.

Here is how you might apply this model. Suppose you ran the regression in Equation 7-17 for a stock, and estimated the following regression coefficients: $a_i = 0.0$, $b_i = 0.9$, $c_i = 0.2$, and $d_i = 0.3$. Assume that the expected market risk premium is 6% (that is, $r_M - r_{RF} = 6\%$) and that the risk-free rate is 6.5%. Suppose the expected value of r_{SMB} is 3.2% and the expected value of r_{HML} is 4.8%.[22] Using the Fama-French three-factor model, the required return is

$$\begin{aligned} r_i &= r_{RF} + a_i + b_i(r_M - r_{RF}) + c_i(r_{SMB}) + d_i(r_{HML}) \\ &= 6.5\% + 0.0\% + 0.9(6\%) + 0.2(3.2\%) + 0.3(4.8\%) \\ &= 13.98\%. \end{aligned} \quad \textbf{(7-18a)}$$

To date, the Fama-French three-factor model has been used primarily by academic researchers rather than by managers at actual companies, the majority of whom are still using the CAPM. Part of this difference was due at one time to the lack of available data. Most professors had access to the type of data required to calculate the factors, but data for the size factor and the B/M factor were not readily available to the general public. Partially to address this problem, Professor French has made the required historical data available at his Web site.[23] However, there is still difficulty in estimating the expected values of the size factor and the B/M factor. Although we know the historical average returns for these factors, we don't know whether the past historical returns are good estimators of the future expected returns for these factors. In other words, we don't know the risk premium associated with the size and book/market sources of risk. Finally, many managers choose to wait and adopt a new theory only after it has been widely accepted by the academic community.

And that isn't the case right now. In fact, there are a number of subsequent studies indicating that the Fama-French model is not correct.[24] Several of these studies suggest that the size effect is no longer having an effect on stock returns, that there never was a size effect (the previous results were caused by peculiarities in the data sources), or that the size effect doesn't apply to most

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[22]These are the average returns Fama and French found in their sample period for r_{SMB} and r_{HML}.

[23]Professor French's web site, **http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html#Research,** now provides time series data for the returns on the factors ($\bar{r}_M - \bar{r}_{RF}$, $\bar{r}_{SMB}$, and $\bar{r}_{HML}$).

[24]See Peter J. Knez and Mark J. Ready, "On the Robustness of Size and Book-to-Market in the Cross-Sectional Regressions," *Journal of Finance*, September 1997, pp. 1355–1382; Dongcheol Kim, "A Reexamination of Firm Size, Book-to-Market, and Earnings Price in the Cross-Section of Expected Stock Returns," *Journal of Financial and Quantitative Analysis*, December 1997, pp. 463–489; Tyler Shumway and Vincent A. Warther, "The Delisting Bias in CRSP's Nasdaq Data and Its Implications for the Size Effect," *Journal of Finance*, December 1999, pp. 2361–2379; Tim Loughran, "Book-to-Market Across Firm Size, Exchange, and Seasonality: Is There an Effect?" *Journal of Financial and Quantitative Analysis*, September 1997, pp. 249–268; and Ilia D. Dichev, "Is the Risk of Bankruptcy a Systematic Risk?" *Journal of Finance*, June 1998, pp. 1131–1147.

companies. Other studies suggest that the book-to-market effect is not as significant as first supposed and that the book-to-market effect is not caused by risk. Another recent study shows that if the composition of a company's assets were changing over time with respect to the mix of physical assets and growth opportunities (such as R&D, patents, etc.), then it would appear as though there were size and book-to-market effects. In other words, even if the returns on the individual assets conform to the CAPM, changes in the mix of assets would cause the firm's beta to change over time in such a way that the firm would appear to have size and book-to-market effects.[25]

SELF-TEST

What are the factors in the Fama-French model?
How can the model be used to estimate the required return on a stock?
Why isn't the model widely used by managers at actual companies?
An analyst has modeled the stock of a company using a Fama-French three-factor model. The risk-free rate is 5%, the required market return is 11%, the risk premium for small stocks (r_{SMB}) is 3.2%, and the risk premium for value stocks (r_{HML}) is 4.8%. If $a_i = 0$, $b_i = 0.7$, $c_i = 1.2$, and $d_i = 0.7$, what is the stock's required return? (16.4%)

7.9 An Alternative Theory of Risk and Return: Behavioral Finance

The Efficient Markets Hypothesis (EMH) is one of the cornerstones of modern finance theory. It implies that, on average, assets trade at prices equal to their intrinsic values. As we note in Chapter 8, the logic behind the EMH is straightforward. If a stock's price is "too low," rational traders will quickly take advantage of this opportunity and will buy the stock. Their actions will quickly push prices back to their equilibrium level. Likewise, if prices are "too high," rational traders will sell the stock, pushing the price down to its equilibrium level. Proponents of the EMH argue that prices cannot be systematically wrong unless you believe that market participants are unable or unwilling to take advantage of profitable trading opportunities.

While the logic behind the EMH is compelling, some events seem to be inconsistent with the EMH. First, there is some evidence that stocks may have short-term momentum. Stocks that perform poorly tend to continue performing poorly over the next 3 to 12 months, and stocks that perform well tend to continue performing well in the short-term future. On the other hand, there is some evidence that stocks have long-term reversals. In particular, stocks that have the lowest returns in a 5-year period tend to outperform the market during the next 5 years. The opposite is true for stocks that outperform the market during a 5-year period: They tend to have lower than average returns during the next 5-year period.[26]

In response to such observations, a number of researchers are blending psychology with finance, creating a new field called **behavioral finance.** There is a large body of evidence in the field of psychology indicating that people don't behave rationally in many areas of their lives, so some argue that we should not expect people to behave rationally with their investments.[27] Pioneers in this field

[25]See Jonathan B. Berk, Richard C. Green, and Vasant Naik, "Optimal Investment, Growth Options, and Security Returns," *Journal of Finance*, October 1999, pp. 1553–1608.
[26]N. Jegadeesh and S. Titman, "Returns to Buying Winners and Selling Losers: Implications for Stock Market Efficiency," *Journal of Finance*, March 1993, pp. 69–91; and W. F. M. DeBondt and R. H. Thaler, "Does the Stock Market Overreact?" *Journal of Finance*, July 1985, pp. 793–808.
[27]See Brian O'Reilly, "Why Johnny Can't Invest," *Fortune*, November 9, 1998, pp. 173–178.

include psychologists Daniel Kahneman and Amos Tversky, along with University of Chicago finance professor Richard Thaler. Their work has encouraged a growing number of scholars to work in this promising area of research.

Professor Thaler and his colleague, Nicholas Barberis, have summarized much of this research in a recent article.[28] They argue that behavioral finance theory rests on two important building blocks. First, they argue that mispricing can persist because it is often difficult or risky for traders to take advantage of mispriced assets. For example, even if it was clear that a stock's price is too low because investors have overreacted to recent bad news, a trader with limited capital may be reluctant to buy the stock for fear that the same forces that pushed the price down may work to keep it artificially low for a long period of time. On the other side, during the recent stock market bubble that burst in 2000, many traders who believed (correctly!) that stock prices were too high lost a lot of money selling stocks in the early stages of the bubble because stock prices went even higher before they eventually collapsed. In other words, there is no safe way to take advantage of mispricing.

While the first building block explains *why* mispricings may persist, the second tries to understand *how* mispricings can occur in the first place. This is where the insights from psychology come into play. For example, Kahneman and Tversky suggested that individuals view potential losses and potential gains very differently. If you ask an average person whether he or she would rather have $500 with certainty or flip a fair coin and receive $1,000 if heads comes up and nothing if it comes out tails, most would prefer the certain $500 gain, which suggests an aversion to risk. However, if you ask the same person whether he or she would rather pay $500 with certainty or flip a coin and pay $1,000 if it's heads and nothing if it's tails, most would indicate that they prefer to flip the coin. But this implies a preference for risk. In other words, people appear to dislike risk when it comes to gains, but will take on risk in order to avoid a sure loss. Other experiments have reinforced this idea that most people experience "loss aversion," or a strong desire to avoid realizing losses. In irrational, but common, mental bookkeeping, a loss isn't really a loss until the losing investment is actually sold. This leads investors to sell losers much less frequently than winners even though this is suboptimal for tax purposes.[29]

Not only do most people view risky gains and losses differently, but other studies suggest that people's willingness to take a gamble depends on recent past performance. Gamblers who are ahead tend to take on more risks (i.e., they are playing with the house's money), whereas those who are behind tend to become more conservative. These experiments suggest that investors and managers behave differently in down markets than they do in up markets, in which they are playing with the "house's" money.

Many psychological tests also show that people are overconfident with respect to their own abilities relative to the abilities of others, which is the basis of Garrison Keillor's joke about a town where all the children are above average. Barberis and Thaler point out that:

> Overconfidence may in part stem from two other biases, self attribution bias and hindsight bias. Self attribution bias refers to people's tendency to ascribe any success they have in some activity to their own talents, while blaming failure on bad luck, rather than on their ineptitude. Doing this repeatedly will lead people to the pleasing but erroneous

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[28]Nicholas Barberis and Richard Thaler, "A Survey of Behavioral Finance," Chapter 18, *Handbook of the Economics of Finance*, edited by George Constantinides, Milt Harris, and René Stulz, part of the Handbooks in Economics Series, Elsevier/North-Holland, 2003.

[29]See Terrance Odean, "Are Investors Reluctant to Realize Their Losses?" *Journal of Finance*, October 1998, pp. 1775–1798.

conclusion that they are very talented. For example, investors might become overconfident after several quarters of investing success [Gervais and Odean (2001)[30]]. Hindsight bias is the tendency of people to believe, after an event has occurred, that they predicted it before it happened. If people think they predicted the past better than they actually did, they may also believe that they can predict the future better than they actually can.

Some researchers have hypothesized that the combination of overconfidence and biased self-attribution leads to overly volatile stock markets, short-term momentum, and long-term reversals.[31] In other words, stock returns reflect the irrational, but predictable, behavior of humans. Behavioral finance also has implications for corporate finance. Recent research by Ulrike Malmendier of the Stanford Graduate School of Business and Geoffrey Tate of the Wharton School suggests that overconfidence leads managers to overestimate their abilities and the quality of their projects.[32] This result may explain why so many corporate projects fail to live up to their stated expectations.

SELF-TEST

What is short-term momentum? What are long-term reversals? What is behavioral finance?

Summary

Chapter 7 completes our discussion of risk and return for traded securities. The primary goal of this chapter was to extend your knowledge of risk and return concepts. The key concepts covered are listed below:

- The **feasible set** of portfolios represents all portfolios that can be constructed from a given set of assets.
- An **efficient portfolio** is one that offers the most return for a given amount of risk, or the least risk for a given amount of return.
- The **optimal portfolio** for an investor is defined by the investor's highest possible **indifference curve** that is tangent to the **efficient set** of portfolios.
- The **Capital Asset Pricing Model (CAPM)** describes the relationship between market risk and required rates of return.
- The **Capital Market Line (CML)** describes the risk/return relationship for efficient portfolios, that is, for portfolios that consist of a mix of the market portfolio and a riskless asset.
- The **Security Market Line (SML)** is an integral part of the CAPM, and it describes the risk/return relationship for individual assets. The required rate of return for any Stock i is equal to the **risk-free rate** plus the **market risk premium** times the stock's **beta coefficient:** $r_i = r_{RF} + (r_M - r_{RF})b_i$.
- Stock i's **beta coefficient, b_i,** is a measure of the stock's **market risk.** Beta measures the **volatility** of returns on a security **relative to returns on the market,** which is the portfolio of all risky assets.

[30]See Terrance Odean and Simon Gervais, "Learning to Be Overconfident," *Review of Financial Studies*, Spring 2001, pp. 1–27.

[31]See Terrance Odean, "Volume, Volatility, Price, and Profit When All Traders Are Above Average," *Journal of Finance*, December 1998, pp. 1887–1934; and Kent Daniel, David Hirshleifer, and Avanidhar Subrahmanyam, "Investor Psychology and Security Market Under- and Overreactions," *Journal of Finance*, December 1998, pp. 1839–1885.

[32]See Ulrike Malmendier and Geoffrey Tate, "CEO Overconfidence and Corporate Investment," *Journal of Finance*, December 2005, pp. 2661–2700.

- The beta coefficient is measured by the slope of the stock's **characteristic line,** which is found by regressing historical returns on the stock versus historical returns on the market.
- Although the CAPM provides a convenient framework for thinking about risk and return issues, it *cannot be proven empirically,* and its parameters are very difficult to estimate. Thus, the required rate of return for a stock as estimated by the CAPM may not be exactly equal to the true required rate of return.
- Deficiencies in the CAPM have motivated theorists to seek other risk/return equilibrium models, and the **Arbitrage Pricing Theory (APT)** is one important new model.
- The **Fama-French three-factor model** has one factor for the **market return,** a second factor for the **size effect,** and a third factor for the **book-to-market effect.**
- **Behavioral finance** assumes that investors don't always behave rationally.

In the next two chapters, we will see how a security's required rate of return affects its value.

Questions

(7-1) Define the following terms, using graphs or equations to illustrate your answers wherever feasible:
a. Portfolio; feasible set; efficient portfolio; efficient frontier
b. Indifference curve; optimal portfolio
c. Capital Asset Pricing Model (CAPM); Capital Market Line (CML)
d. Characteristic line; beta coefficient, b
e. Arbitrage Pricing Theory (APT); Fama-French three-factor model; behavioral finance

(7-2) Security A has an expected rate of return of 6%, a standard deviation of returns of 30%, a correlation coefficient with the market of −0.25, and a beta coefficient of −0.5. Security B has an expected return of 11%, a standard deviation of returns of 10%, a correlation with the market of 0.75, and a beta coefficient of 0.5. Which security is more risky? Why?

Self-Test Problem Solution Appears in Appendix A

(ST-1) Risk and Return

You are planning to invest $200,000. Two securities, A and B, are available, and you can invest in either of them or in a portfolio with some of each. You

estimate that the following probability distributions of returns are applicable for A and B:

Security A		Security B	
P_A	r_A	P_B	r_B
0.1	−10%	0.1	−30%
0.2	5	0.2	0
0.4	15	0.4	20
0.2	25	0.2	40
0.1	40	0.1	70
	$\hat{r}_A$ = ?		$\hat{r}_B$ = 20.0%
	σ_A = ?		σ_B = 25.7%

a. The expected return for Security B is $\hat{r}_B = 20\%$, and $\sigma_B = 25.7\%$. Find $\hat{r}_A$ and σ_A.
b. Use Equation 7-3 to find the value of w_A that produces the minimum risk portfolio. Assume $\rho_{AB} = -0.5$ for parts b and c.
c. Construct a table giving $\hat{r}_p$ and σ_p for portfolios with $w_A = 1.00, 0.75, 0.50, 0.25, 0.0$, and the minimum risk value of w_A. (Hint: For $w_A = 0.75$, $\hat{r}_p = 16.25\%$ and $\sigma_p = 8.5\%$; for $w_A = 0.5$, $\hat{r}_p = 17.5\%$ and $\sigma_p = 11.1\%$; for $w_A = 0.25$, $\hat{r}_p = 18.75\%$ and $\sigma_p = 17.9\%$.)
d. Graph the feasible set of portfolios and identify the efficient frontier of the feasible set.
e. Suppose your risk/return trade-off function, or indifference curve, is tangent to the efficient set at the point where $\hat{r}_p = 18\%$. Use this information, plus the graph constructed in part d, to locate (approximately) your optimal portfolio. Draw in a reasonable indifference curve, indicate the percentage of your funds invested in each security, and determine the optimal portfolio's σ_p and $\hat{r}_p$. (Hint: Estimate σ_p and $\hat{r}_p$ graphically, and then use the equation for $\hat{r}_p$ to determine w_A.)
f. Now suppose a riskless asset with a return $\hat{r}_{RF} = 10\%$ becomes available. How would this change the investment opportunity set? Explain why the efficient frontier becomes linear.
g. Given the indifference curve in part e, would you change your portfolio? If so, how? (Hint: Assume the indifference curves are parallel.)
h. What are the beta coefficients of Stocks A and B? [Hints: (1) Recognize that $r_i = r_{RF} + b_i(r_M - r_{RF})$ and solve for b_i, and (2) assume that your preferences match those of most other investors.]

Problems Answers Appear in Appendix B

Easy Problems 1–3

(7-1) Beta
The standard deviation of stock returns for Stock A is 40%. The standard deviation of the market return is 20%. If the correlation between Stock A and the market is 0.70, what is Stock A's beta?

(7-2) APT
An analyst has modeled the stock of Crisp Trucking using a two-factor APT model. The risk-free rate is 6%, the expected return on the first factor (r_1) is 12%, and the expected return on the second factor (r_2) is 8%. If $b_{i1} = 0.7$ and $b_{i2} = 0.9$, what is Crisp's required return?

(7-3) Fama-French Three-Factor Model

An analyst has modeled the stock of a company using a Fama-French three-factor model. The risk-free rate is 5%, the required market return is 10%, the risk premium for small stocks (r_{SMB}) is 3.2%, and the risk premium for value stocks (r_{HML}) is 4.8%. If $a_i = 0$, $b_i = 1.2$, $c_i = -0.4$, and $d_i = 1.3$, what is the stock's required return?

Intermediate Problems 4–6

(7-4) Two-Asset Portfolio

Stock A has an expected return of 12% and a standard deviation of 40%. Stock B has an expected return of 18% and a standard deviation of 60%. The correlation coefficient between Stocks A and B is 0.2. What are the expected return and standard deviation of a portfolio invested 30% in Stock A and 70% in Stock B?

(7-5) SML and CML Comparison

The beta coefficient of an asset can be expressed as a function of the asset's correlation with the market as follows:

$$b_i = \frac{\rho_{i,M}\sigma_i}{\sigma_M}.$$

a. Substitute this expression for beta into the Security Market Line (SML), Equation 7-9. This results in an alternative form of the SML.
b. Compare your answer to part a with the Capital Market Line (CML), Equation 7-6. What similarities are observed? What conclusions can be drawn?

(7-6) CAPM and the Fama-French Three-Factor Model

Suppose you are given the following information. The beta of company i, b_i, is 1.1, the risk-free rate, r_{RF}, is 7%, and the expected market premium, $r_M - r_{RF}$, is 6.5%. (Assume that $a_i = 0.0$.)

a. Use the Security Market Line (SML) of CAPM to find the required return for this company.
b. Because your company is smaller than average and more successful than average (that is, it has a low book-to-market ratio), you think the Fama-French three-factor model might be more appropriate than the CAPM. You estimate the additional coefficients from the Fama-French three-factor model: The coefficient for the size effect, c_i, is 0.7, and the coefficient for the book-to-market effect, d_i, is −0.3. If the expected value of the size factor is 5% and the expected value of the book-to-market factor is 4%, what is the required return using the Fama-French three-factor model?

Challenging Problems 7–8

(7-7) Characteristic Line and Security Market Line

You are given the following set of data:

	Historical Rates of Return	
Year	NYSE	Stock X
1	(26.5%)	(14.0%)
2	37.2	23.0
3	23.8	17.5
4	(7.2)	2.0
5	6.6	8.1
6	20.5	19.4
7	30.6	18.2

a. Use a spreadsheet (or a calculator with a linear regression function) to determine Stock X's beta coefficient.

b. Determine the arithmetic average rates of return for Stock X and the NYSE over the period given. Calculate the standard deviations of returns for both Stock X and the NYSE.
c. Assuming (1) that the situation during Years 1 to 7 is expected to hold true in the future (that is, $\hat{r}_X = \bar{r}_X$; $\hat{r}_M = \bar{r}_M$; and both σ_X and b_X in the future will equal their past values), and (2) that Stock X is in equilibrium (that is, it plots on the Security Market Line), what is the risk-free rate?
d. Plot the Security Market Line.
e. Suppose you hold a large, well-diversified portfolio and are considering adding to the portfolio either Stock X or another stock, Stock Y, that has the same beta as Stock X but a higher standard deviation of returns. Stocks X and Y have the same expected returns; that is, $\hat{r}_X = \hat{r}_Y = 10.6\%$. Which stock should you choose?

(7-8) Characteristic Line

You are given the following set of data:

	Historical Rates of Return	
Year	NYSE	Stock Y
1	4.0%	3.0%
2	14.3	18.2
3	19.0	9.1
4	(14.7)	(6.0)
5	(26.5)	(15.3)
6	37.2	33.1
7	23.8	6.1
8	(7.2)	3.2
9	6.6	14.8
10	20.5	24.1
11	30.6	18.0
	Mean = 9.8%	9.8%
	σ = 19.6%	13.8%

a. Construct a scatter diagram showing the relationship between returns on Stock Y and the market. Use a spreadsheet or a calculator with a linear regression function to estimate beta.
b. Give a verbal interpretation of what the regression line and the beta coefficient show about Stock Y's volatility and relative risk as compared with those of other stocks.
c. Suppose the scatter of points had been more spread out, but the regression line was exactly where your present graph shows it. How would this affect (1) the firm's risk if the stock is held in a one-asset portfolio and (2) the actual risk premium on the stock if the CAPM holds exactly?
d. Suppose the regression line had been downward sloping and the beta coefficient had been negative. What would this imply about (1) Stock Y's relative risk, (2) its correlation with the market, and (3) its probable risk premium?

Spreadsheet Problem

(7-9) Feasible Portfolios

Start with the partial model in the file ***FM12 Ch 07 P09 Build a Model.xls*** from the textbook's Web site. Following is information for the required returns and standard deviations of returns for A, B, and C.

FM e-resource

Stock	r_i	σ_i
A	7.0%	33.11%
B	10.0%	53.85%
C	20.0%	89.44%

The correlation coefficients for each pair are shown below in a matrix, with each cell in the matrix showing the correlation between the stock in the same row and column. For example, $\rho_{AB} = 0.1571$ is in the row for A and the column for B. Notice that the diagonal values are equal to 1, because a variable is always perfectly positively correlated with itself.

	A	B	C
A	1.0000	0.1571	0.1891
B	0.1571	1.0000	0.1661
C	0.1891	0.1661	1.0000

a. Suppose a portfolio has 30% invested in A, 50% in B, and 20% in C. What are the expected return and standard deviation of the portfolio?
b. The partial model lists 66 different combinations of portfolio weights. For each combination of weights, find the required return and standard deviation.
c. The partial model provides a scatter diagram showing the required returns and standard deviations calculated above. This provides a visual indicator of the feasible set. If you would like a return of 10.50%, what is the smallest standard deviation that you must accept?

Cyberproblem

FM e-resource

Please go to the textbook's Web site to access any Cyberproblems.

Mini Case

To begin, briefly review the Chapter 6 Mini Case. Then, extend your knowledge of risk and return by answering the following questions:

a. Suppose Asset A has an expected return of 10% and a standard deviation of 20%. Asset B has an expected return of 16% and a standard deviation of 40%. If the correlation between A and B is 0.35, what are the expected return and standard deviation for a portfolio comprised of 30% Asset A and 70% Asset B?

b. Plot the attainable portfolios for a correlation of 0.35. Now plot the attainable portfolios for correlations of +1.0 and −1.0.

c. Suppose a risk-free asset has an expected return of 5%. By definition, its standard deviation is zero, and its correlation with any other asset is also zero. Using only Asset A and the risk-free asset, plot the attainable portfolios.

d. Construct a reasonable, but hypothetical, graph that shows risk, as measured by portfolio standard deviation, on the x-axis and expected rate of return on the y-axis. Now add an illustrative feasible (or attainable) set of portfolios, and show what portion of the feasible set is efficient. What makes a particular portfolio efficient? Don't worry about specific values when constructing the graph—merely illustrate how things look with "reasonable" data.

e. Now add a set of indifference curves to the graph created for part b. What do these curves represent? What is the optimal portfolio for this investor? Finally, add a second set of indifference curves that leads to the selection of a different optimal portfolio. Why do the two investors choose different portfolios?

f. What is the Capital Asset Pricing Model (CAPM)? What are the assumptions that underlie the model?

g. Now add the risk-free asset. What impact does this have on the efficient frontier?

h. Write out the equation for the Capital Market Line (CML) and draw it on the graph. Interpret the CML. Now add a set of indifference curves, and illustrate how an investor's optimal portfolio is some combination of the risky portfolio and the risk-free asset. What is the composition of the risky portfolio?

i. What is a characteristic line? How is this line used to estimate a stock's beta coefficient? Write out and explain the formula that relates total risk, market risk, and diversifiable risk.

j. What are two potential tests that can be conducted to verify the CAPM? What are the results of such tests? What is Roll's critique of CAPM tests?

k. Briefly explain the difference between the CAPM and the Arbitrage Pricing Theory (APT).

l. Suppose you are given the following information. The beta of a company, b_i, is 0.9; the risk-free rate, r_{RF}, is 6.8%; and the expected market premium, $r_M - r_{RF}$, is 6.3%. Because your company is larger than average and more successful than average (that is, it has a lower book-to-market ratio), you think the Fama-French three-factor model might be more appropriate than the CAPM. You estimate the additional coefficients from the Fama-French

three-factor model: The coefficient for the size effect, c_i, is −0.5, and the coefficient for the book-to-market effect, d_i, is −0.3. If the expected value of the size factor is 4% and the expected value of the book-to-market factor is 5%, what is the required return using the Fama-French three-factor model? (Assume that $a_i = 0.0$.) What is the required return using CAPM?

Selected Additional Cases

The following case from Textchoice, *Thomson Learning's online library, covers many of the concepts discussed in this chapter and is available at* ***http://www.textchoice2.com.***

Klein-Brigham Series:
Case 2, "Peachtree Securities, Inc. (A)."